MATHEMATICS (PART–II)

SOLUTION : PRACTICE QUESTION PAPER 5

Q. 1. (A) (i) (C)

(ii) (D)

(iii) (C)

(iv) (D)

Q. 1. (A) Explanation to the answers to MCQs in this question has been given below for students' guidance. Please note that, **Students are not expected to write the explanation in the examination**.

Explanations :

(i) (C) [$1+\cot^2\theta = \text{cosec}^2\theta$ (Trigonometric identity]

(ii) (D) [Curved surface area of right circular cone $= \pi rl$]

(iii) (C) [When two circles touch externally, then three tangents can be drawn.]

(iv) (D) [Height of equilateral triangle $= \frac{\sqrt{3}S}{2}$]

Q. 1. (B) **(i) Ans.** $\triangle ABC \sim \triangle PQR$ by SSS test of similarity.

(ii) Solution : $4^2+5^2 = 16+25 = 41$

$8^2 = 64$

Here $4^2+5^2 \neq 8^2$

Ans. (4, 5, 8) is not a Pythagorean triplet.

(iii) Ans. Tangent segments are congruent.

$\therefore$ PR = PQ $\quad \therefore$ PR = 5 cm.

(iv) Solution : $\cos(45°+x) = \sin 30°$

$\cos(45°+x) = \frac{1}{2}$ $\quad ... \left(\sin 30° = \frac{1}{2}\right)$

We know $\cos 60° = \frac{1}{2}$

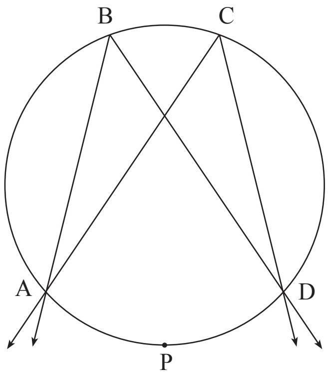
$\therefore \cos(45°+x) = \cos 60°$

$\therefore 45+x = 60 \quad \therefore x = 60-45$

$\therefore x = 15°$

Ans. The value of x is **15°**.

Q. 2. (A) (i)



Activity :

$\angle ABD = \frac{1}{2} m(\text{arc } \boxed{\textbf{APD}})$ … (1)

(Inscribed angle theorem)

$\angle ACD = \frac{1}{2} m(\text{arc } \boxed{\textbf{APD}})$ … (2)

$\therefore$ from (1) and (2)

$\angle ABD = \boxed{\angle \textbf{ACD}}$

$\therefore$ angles inscribed in the same arc are $\boxed{\textbf{congruent}}$.

(ii) Activity :

Let P (x_1, y_1) and Q (x_2, y_2)

$x_1 = -6, y_1 = -3, x_2 = -1$ and $y_2 = 9$

$PQ = \sqrt{\boxed{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$ … (Distance formula)

$\therefore PQ = \sqrt{\boxed{\textbf{25}} + 144}$

$\therefore PQ = \sqrt{\boxed{\textbf{169}}}$

$\therefore PQ = \boxed{\textbf{13}}$.

(iii) Activity :

Radius of the sphere, $r = 18$ cm.

For cylinder, radius R = 6 cm, height H = 12 cm

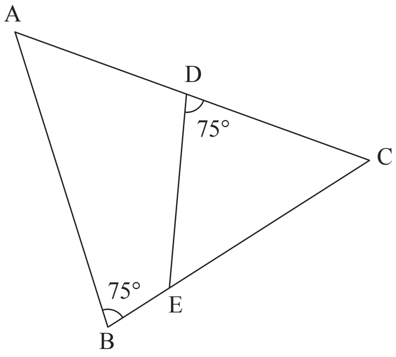
No. of cylinders can be made $= \frac{\text{Volume of the sphere}}{\boxed{\textbf{Volume of each cylinder}}}$

$= \frac{\frac{4}{3}\pi r^3}{\boxed{\pi \mathbf{R^2H}}}$

$$= \frac{\frac{4}{3} \times 18 \times 18 \times 18}{\boxed{\mathbf{6 \times 6 \times 12}}}$$

$$= \boxed{\mathbf{18}}.$$

Q. 2. (B) (i)



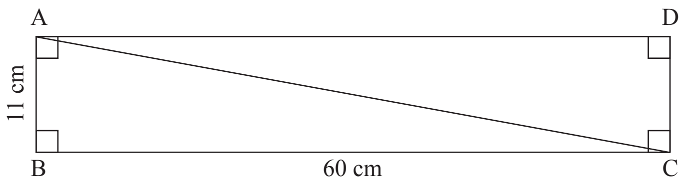
Proof : In $\triangle DCE$ and $\triangle BCA$,

$\angle CDE \cong \angle CBA$ ... (Each measures 75°)

$\angle DCE \cong \angle BCA$ ... (Common angle)

$\therefore \triangle DCE \sim \triangle BCA$ ... (AA test of similarity)

(ii) Solution :



Let $\square$ABCD be given rectangle.

AB = 11 cm, BC = 60 cm.

In $\triangle ABC$,

$\angle ABC = 90°$ ... (Angle of a rectangle)

$\therefore$ by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = 11^2 + 60^2$

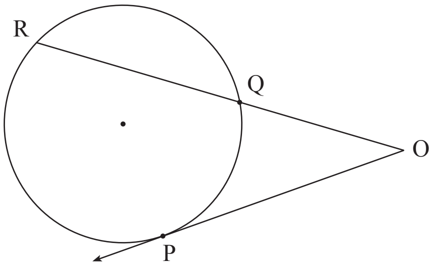
$\therefore AC^2 = 121 + 3600$

$\therefore AC^2 = 3721$

$\therefore AC = 61$ cm ... (Taking square roots of both the sides)

Ans. Length of diagonal of a rectangle is **61 cm.**

(iii)



Solution : Line OP is tangent to the circle at point P and line OQR is secant to the circle intersecting the circle at points Q and R.

$\therefore$ by tangent secant segments theorem,

$OP^2 = OQ \times OR$

$\therefore\ 7.2^2 = 3.2 \times OR$

$\therefore\ OR = \dfrac{7.2 \times 7.2}{3.2}$

$\therefore\ OR = 16.2$

$OR = OQ + QR$

$\therefore\ 16.2 = 3.2 + QR$

$\therefore\ QR = 16.2 - 3.2 = 13$

Ans. QR = **13**.

(iv) Solution : R(0, 3), D(2, 1) and S(3, −1).

By distance formula,

$$\begin{aligned} d(R, D) &= \sqrt{(2-0)^2 + (1-3)^2} \\ &= \sqrt{(2)^2 + (-2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= \sqrt{2 \times 2 \times 2} \\ &= 2\sqrt{2} \quad \text{... (1)} \end{aligned}$$

$$\begin{aligned} d(D, S) &= \sqrt{(3-2)^2 + (-1-1)^2} \\ &= \sqrt{1^2 + (-2)^2} \\ &= \sqrt{1+4} \\ &= \sqrt{5} \quad \text{... (2)} \end{aligned}$$

$$\begin{aligned} d(R, S) &= \sqrt{(3-0)^2 + (-1-3)^2} \\ &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \quad \text{... (3)} \end{aligned}$$

Adding (2) and (3),

$d(\text{D, S}) + d(\text{R, S}) = \sqrt{5} + 5$ ... (4)

$\therefore$ $d(\text{D, S}) + d(\text{R, S}) \neq d(\text{R, D})$ ... [From (1) and (4)]

$\therefore$ points R, D and S are not collinear.

Ans. Points R(0, 3), D(2, 1) and S(3, − 1) **are not collinear.**

(v) Solution :

$\sin\theta = \frac{7}{25}$

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \left(\frac{7}{25}\right)^2 + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \frac{49}{625}$

$\therefore \cos^2\theta = \frac{625 - 49}{625}$

$\therefore \cos^2\theta = \frac{576}{625}$ … (Taking square root of both the sides)

$\therefore \cos\theta = \frac{24}{25}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

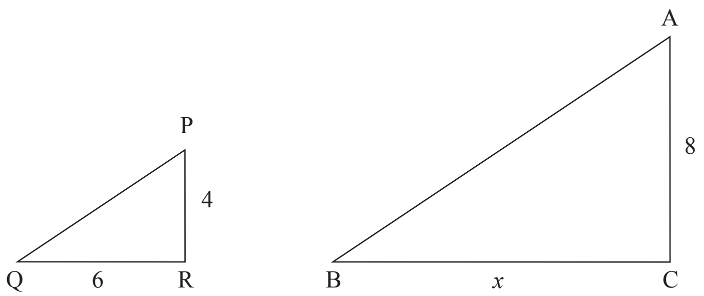
$= \frac{7}{25} \div \frac{24}{25}$

$= \frac{7}{\cancel{25}} \times \frac{\cancel{25}}{24}$

$= \frac{7}{24}$

Ans. The value of $\cos\theta$ is $\mathbf{\frac{24}{25}}$ and $\tan\theta$ is $\mathbf{\frac{7}{24}}$.

Q. 3. (A) (i)



Activity :

Let BC $= x$

The shadows are cast at the same time

$\triangle$PQR $\sim$ $\triangle$ **ABC**

$\therefore \dfrac{PR}{\boxed{AC}} = \dfrac{QR}{BC}$ … (Corresponding sides of similar triangles are in proportion)

$\therefore \dfrac{4}{\boxed{8}} = \dfrac{6}{x}$

$\therefore 4 \times x = \boxed{8} \times 6$

$\therefore x = \dfrac{\boxed{8} \times 6}{4}$

$\therefore x = \boxed{12}$

Ans. Length of the shadow of bigger pole is 12 m.

(ii) Activity :

P $(-2, 2)$, Q $(2, 2)$ and R $(2, 7)$

Using distance formula,

$PQ = \boxed{4}$ … (1)

$QR = \boxed{5}$ … (2)

$PR = \boxed{\sqrt{41}}$ … (3)

$PQ^2 + QR^2 = \boxed{41}$ … [Using the values obtained in (1) and (2)] … (4)

$PR^2 = \boxed{41}$ … [Using the value obtained in (3)] … (5)

$\therefore$ from (4) and (5)

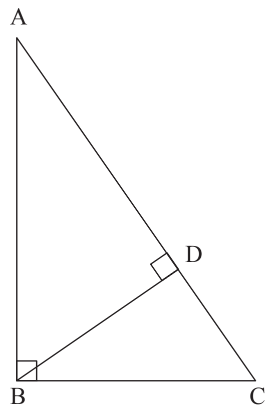
$PQ^2 + QR^2 = \boxed{PR^2}$

$\therefore$ by converse of Pythagoras theorem,

$\triangle PQR$ is a right angled triangle.

i.e. P$(-2, 2)$, Q$(2, 2)$ and R$(2, 7)$ are the vertices of a right angled triangle.

Q. 3. (B) (i)



Given : In $\triangle ABC$, $\angle ABC = 90°$.

seg BD $\perp$ hypotenuse AC such that A–D–C.

To prove : $BD^2 = AD \times DC$.

Proof : In $\triangle ABC$, $\angle ABC = 90°$ ... (Given)

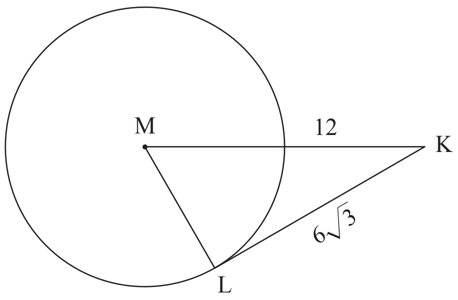
seg BD $\perp$ hypotenuse AC

$\therefore \triangle ADB \sim \triangle BDC$ ... (Similarity of right angled triangles)

$\therefore \dfrac{AD}{BD} = \dfrac{BD}{DC}$ ... (Corresponding sides of similar triangles are in proportion)

$\therefore BD^2 = AD \times CD$.

(ii)



Solution : In $\triangle MLK$,

$\angle MLK = 90°$ ... (Tangent theorem)

$\therefore$ by Pythagoras theorem,

$MK^2 = ML^2 + LK^2$

$\therefore 12^2 = ML^2 + (6\sqrt{3})^2$

$\therefore 144 = ML^2 + 36 \times 3$

$\therefore 144 = ML^2 + 108$

$\therefore ML^2 = 144 - 108$

$\therefore ML^2 = 36$

$\therefore ML = 6$ ... (Taking square roots of both the sides)

$\therefore$ radius of the circle $= ML = 6$.

In $\triangle MLK$,

$ML = \dfrac{1}{2} MK$

$\therefore \angle K = 30°$ ... (By converse of 30°–60°–90° triangle theorem)

In $\triangle MLK$,

$\angle M + \angle K + \angle L = 180°$ ... (Sum of all angles of a triangle is 180°)

$\therefore \angle M + 30° + 90° = 180°$

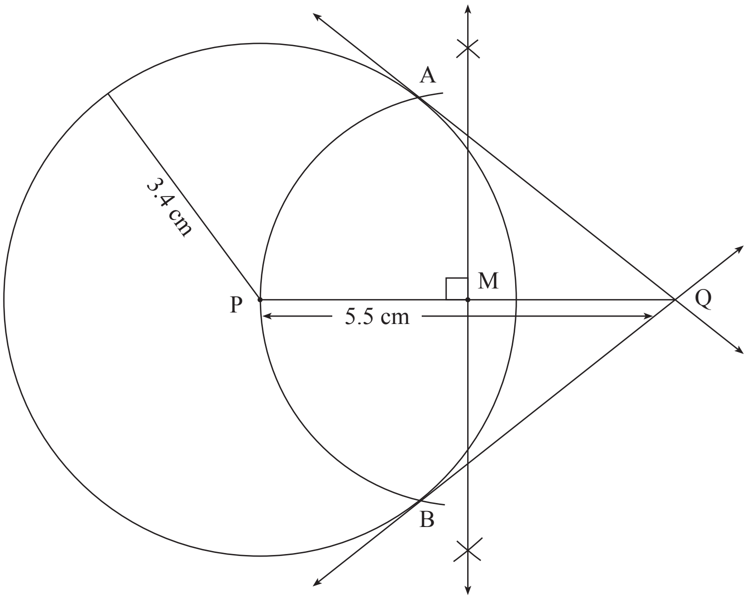
$\therefore \angle M + 120° = 180°$

$\therefore \angle M = 180° - 120°$

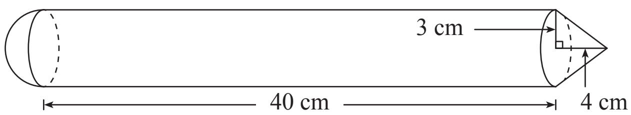
$\therefore \angle M = 60°$

Ans. (1) Radius of the circle is **6** (2) $\angle K =$ **30°** and $\angle M =$ **60°**.

(iii)



(iv)



Solution : A toy is made up of hemisphere, cylinder and a cone.

They have equal bases.

Let their radius be r.

$\therefore r = 3$ cm.

Let the heights of the conical part and cylindrical part be h_1 and h_2 respectively.

$h_1 = 4$ cm and $h_2 = 40$ cm.

Let the slant height of the cone be l.

$l^2 = r^2 + h_1^2$

$\therefore l^2 = 3^2 + 4^2$

$\therefore l^2 = 9 + 16$

$\therefore l^2 = 25$

$\therefore l = 5$ cm ... (Taking square roots of both the sides)

Total area of the toy = curved surface area of the cone + curved surface area of the cylinder + curved surface area of the hemisphere

$= \pi rl + 2\pi rh_2 + 2\pi r^2$

$= \pi r(l + 2h_2 + 2r)$

$= \pi \times 3(5 + 2 \times 40 + 2 \times 3)$

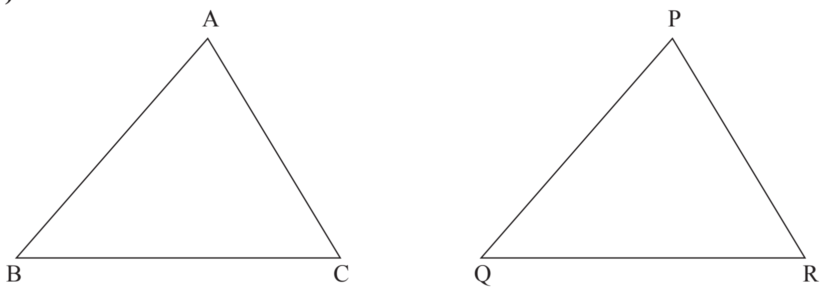
$= \pi \times 3(5 + 80 + 6)$

$= \pi \times 3(91)$

$= 273\pi \text{ cm}^2$

Ans. Total area of the toy is $\mathbf{273\pi \text{ cm}^2}$.

Q. 4. (i)



Given : $\triangle ABC \sim \triangle PQR$

$A(\triangle ABC) = A(\triangle PQR)$

To prove : $\triangle ABC \cong \triangle PQR$

Proof : $A(\triangle ABC) = A(\triangle PQR)$ … (Given)

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle PQR)} = 1$ … (1)

$\triangle ABC \sim \triangle PQR$ … (Given)

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$ … (Areas of similar triangles)

$\therefore 1 = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$ … [From (1)]

$\therefore \dfrac{AB^2}{PQ^2} = 1$	$\therefore \dfrac{BC^2}{QR^2} = 1$	$\therefore \dfrac{AC^2}{PR^2} = 1$
$\therefore AB^2 = PQ^2$	$\therefore BC^2 = QR^2$	$\therefore AC^2 = PR^2$
$\therefore AB = PQ$ … (2)	$\therefore BC = QR$ … (3)	$\therefore AC = PR$ … (4)

In $\triangle ABC$ and $\triangle PQR$,

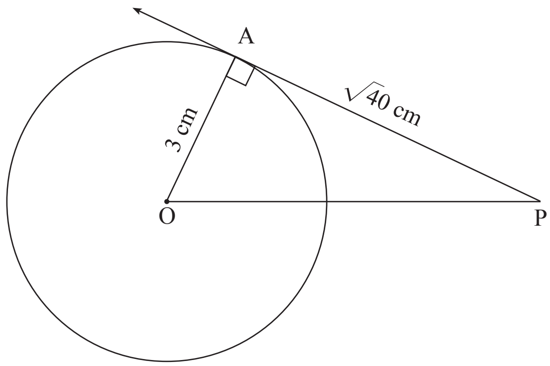
side $AB =$ side PQ … [From (2)]

side $BC =$ side QR … [From (3)]

side $AC =$ side PR … [From (4)]

$\therefore \triangle ABC \cong \triangle PQR$ … (SSS test of congruence)

(ii) Analysis :



$\angle OAP = 90°$ ... (Tangent theorem)

$\triangle OAP$ is a right angled triangle

So, by Pythagoras theorem, the length of segment OP can be determined

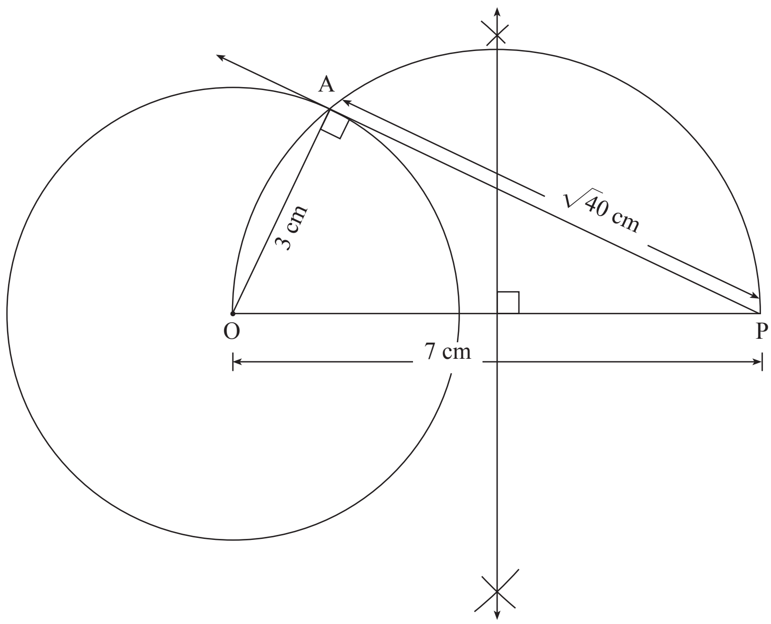
OP = 7 cm

Thus location of P can be determined.

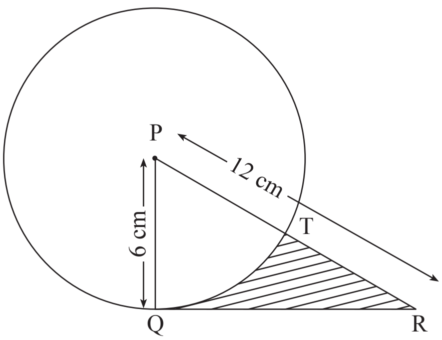
From external point P, tangent can be drawn to the circle.

The length of tangent would be $\sqrt{40}$ cm.

Ans.



(iii)



Solution :

$\angle PQR = 90°$ … (Tangent theorem)

In right angled $\triangle PQR$,

$$\cos P = \frac{PQ}{PR} = \frac{6}{12} = \frac{1}{2}$$

We know, $\cos 60° = \frac{1}{2}$

$\therefore \angle P = 60°$

$$\sin 60° = \frac{QR}{PR}$$

$$\therefore \frac{\sqrt{3}}{2} = \frac{QR}{12}$$

$$\therefore QR = \frac{12\sqrt{3}}{2}$$

$$\therefore QR = 6\sqrt{3}$$

$$A(\triangle PQR) = \frac{1}{2} \times QR \times PQ$$

$$= \frac{1}{\not{2}} \times 6\sqrt{3} \times \not{6}^{3}$$

$$= 18\sqrt{3}$$

$$= 18 \times 1.73$$

$$= 31.14 \text{ cm}^2$$

For sector P–QT,

Angular measure of arc QT $(\theta) = \angle P = 60°$

radius of the circle $= PQ = 6$ cm

$$A(\text{sector P–QT}) = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times 3.14 \times 6 \times 6$$

$$= 3.14 \times 6$$

$$= 18.84 \text{ sq cm}$$

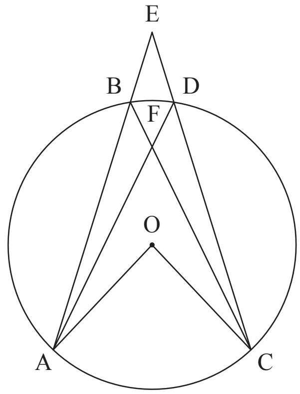
$$A(\text{shaded portion}) = A(\triangle PQR) - A(\text{Sector P–QT})$$

$$= 31.14 - 18.84$$

$$= 12.30 \text{ sq cm}$$

Ans. Area of shaded portion is **12.30 cm²**.

Q. 5. **(i) Ans.**



(a) $\angle AFC = \frac{1}{2}[m(\text{arc AC}) + m(\text{arc BD})]$

(b) $\angle AEC = \frac{1}{2}[m(\text{arc AC}) - m(\text{arc BD})]$

(c) **Proof :** $\angle AOC = m(\text{arc AC})$ … (Definition of measure of minor arc) … (1)

$$\angle AFC + \angle AEC = \frac{1}{2}[m(\text{arc AC}) + m(\text{arc BD})] + \frac{1}{2}[m(\text{arc AC}) - m(\text{arc BD})]$$

$$= \frac{1}{2}[m(\text{arc AC}) + \cancel{m(\text{arc BD})} + m(\text{arc AC}) - \cancel{m(\text{arc BD})}]$$

$$= \frac{1}{2} \times [2m(\text{arc AC})]$$

$$= \frac{1}{\cancel{2}} \times \cancel{2}\,[m(\text{arc AC})]$$

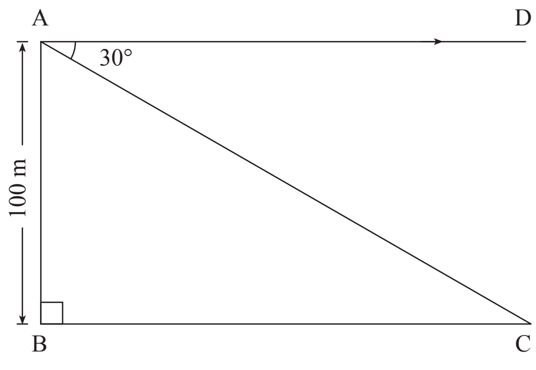
$$= m(\text{arc AC})$$

$$= \angle AOC \quad \text{… [From (1)]}$$

Thus, $\angle AOC = \angle AFC + \angle AEC$.

(ii) (a) $\angle ACB = 30°$ … (Alternate interior angles)

(b) In right angled $\triangle ABC$,



$\tan \angle ACB = \frac{AB}{BC}$

$\tan 30° = \frac{100}{BC}$

$\frac{1}{\sqrt{3}} = \frac{100}{BC}$

$\therefore BC = 100\sqrt{3}$ m

Ans. (a) The measure of $\angle ACB$ is **30°, alternate interior angles.**

(b) Distance between the ship and the light house is **100 $\sqrt{3}$ m**.