MATHEMATICS (PART–II)

SOLUTION : PRACTICE QUESTION PAPER 4

Q. 1. (A) (i) (B)

(ii) (B)

(iii) (D)

(iv) (A)

Q. 1. (A) Explanation to the answers to MCQs in this question has been given below for students' guidance. Please note that, **Students are not expected to write the explanation in the examination**.

Explanations :

(i) (B) Areas of triangles with equal heights are proportional to their corresponding bases.

$\therefore \dfrac{A(\triangle ABD)}{A(\triangle ADC)} = \dfrac{BD}{DC}$

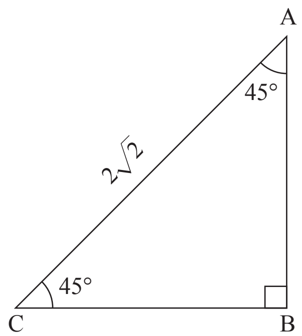
(ii) (B) [In cyclic $\square$ABCD, $\angle A + \angle C = 180°$, $2\angle A = 3\angle C$

$\therefore 2\angle A - 3\angle C = 0$. Solve two equations]

(iii) (D) $\left[\text{Slope of line} = \dfrac{y_2 - y_1}{x_2 - x_1}\right]$

(iv) (A) [Volume of cube $= (\text{side})^3$].

Q. 1. (B) (i) Solution :



$\triangle$ABC is a $45° - 45° - 90°$ triangle

$\therefore$ by $45° - 45° - 90°$ triangle theorem,

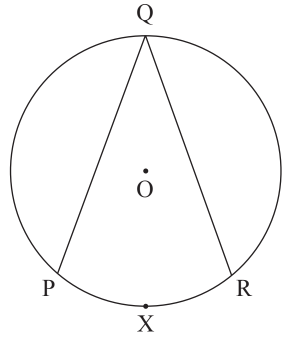
$AB = \dfrac{1}{\sqrt{2}} AC$ … (Side opposite to 45°)

$\therefore AB = \dfrac{1}{\sqrt{2}} \times 2\sqrt{2}$

$\therefore AB = 2$

Ans. AB = **2**.

(ii) Solution :



$m(\text{arc PXR}) = 80°$

$\angle PQR = \frac{1}{2} \times m(\text{arc PXR})$ … [Inscribed angle theorem]

$= \frac{1}{2} \times 80° = 40°$

Ans. Measure of $\angle PQR$ is **40°**.

(iii) Ans.

The longest chord of the circle is 8 cm.

Diameter is the longest chord of the circle.

Radius $= \frac{1}{2} \times$ diameter

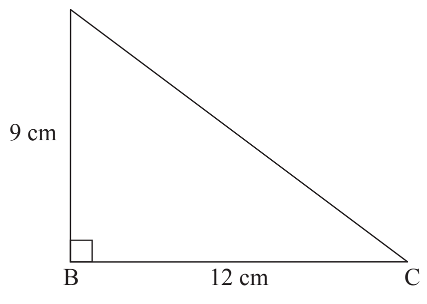
$= \frac{1}{2} \times 8$

$= 4$ cm

The radius of the circle is 4 cm.

(iv) Ans. The point of intersection of line $x = 2$ and $y = -3$ is **(2, −3)**.

Q. 2. (A) (i)



Activity :

$\triangle ABC$ is a right angled triangle,

AB = 9 cm, BC = 12 cm

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 9^2 + \boxed{\mathbf{12^2}}$$

$$= 81 + \boxed{\mathbf{144}} = \boxed{\mathbf{225}}$$

$\therefore$ AC = $\boxed{\mathbf{15}}$ cm.

(ii) Activity :

$$\text{LHS} = \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$$

$$= \frac{1+\sin\theta + \boxed{\mathbf{(1-\sin\theta)}}}{(1-\sin\theta)\,(1+\sin\theta)}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{1-\boxed{\mathbf{\sin^2\theta}}}$$

$$= \frac{2}{\boxed{\mathbf{\cos^2\theta}}} = 2 \times \frac{1}{\cos^2\theta}$$

$$= 2\ \boxed{\mathbf{\sec^2\theta}}$$

$$= \text{RHS}.$$

(iii) Activity :

The radius of the sector $(r) = 3.5$ cm

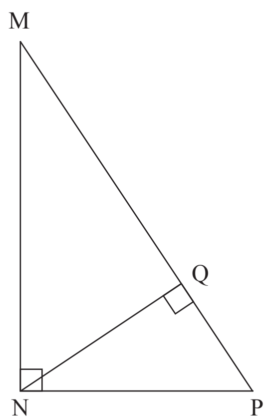
Length of the arc $= 2.2$ cm

$$\text{Area of the sector} = \frac{\text{Length of the arc} \times \boxed{\textbf{radius}}}{2}$$

$$= \frac{\boxed{\mathbf{2.2}} \times \boxed{\mathbf{3.5}}}{2}$$

$$= \boxed{\mathbf{3.85}}\ \text{cm}^2.$$

Q. 2. (B) Solution :



(i) In $\triangle$MNP,

$\angle MNP = 90°$

$NQ \perp MP$

$\therefore$ by theorem of geometric mean,

$NQ^2 = MQ \times QP$

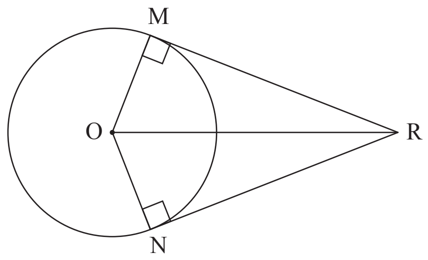
$\therefore\ NQ^2 = 9 \times 4$

$\therefore\ NQ^2 = 36$

$\therefore\ NQ = 6$

Ans. NQ = **6**.

(ii)



Proof : In $\triangle$RMO and $\triangle$RNO

$\angle RMO = \angle RNO = 90°$ ... (Tangent theorem)

Hypotenuse OR $\cong$ hypotenuse OR ... (Common side)

Side OM $\cong$ side ON ... (Radii of the same circle)

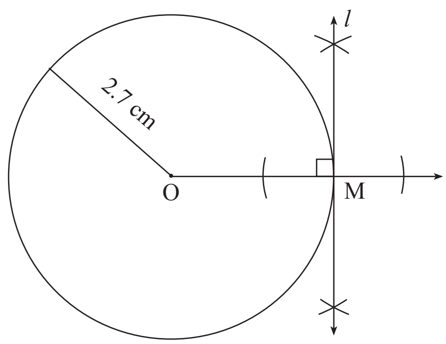
$\therefore\ \triangle RMO \cong \triangle RNO$ ... (Hypotenuse side theorem)

$\therefore\ \angle MRO \cong \angle NRO$

and $\angle MOR \cong \angle NOR$ ... (c.a.c.t.)

$\therefore$ seg OR bisects $\angle$MRN and $\angle$MON.

(iii)



(iv) Solution : P(− 12, − 3) and Q(4, k).

Let P(x_1, y_1) and Q(x_2, y_2).

Here, $x_1 = -12$, $y_1 = -3$, $x_2 = 4$ and $y_2 = k$.

Slope of the line $= m = \frac{1}{2}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore\ \frac{1}{2} = \frac{k - (-3)}{4 - (-12)}$

$\therefore \dfrac{1}{2} = \dfrac{k+3}{4+12}$

$\therefore \dfrac{1}{2} = \dfrac{k+3}{16}$

$\therefore 2(k+3) = 16$

$\therefore (k+3) = \dfrac{16}{2}$ $\quad \therefore k+3 = 8$

$\therefore k = 8 - 3$

$\therefore k = 5$

Ans. $k = \mathbf{5}$**.**

(v) Proof :

$$\text{LHS} = \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{(\sec\theta - \tan\theta)} \times \frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}$$

$$= \frac{1\,(\sec\theta + \tan\theta)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

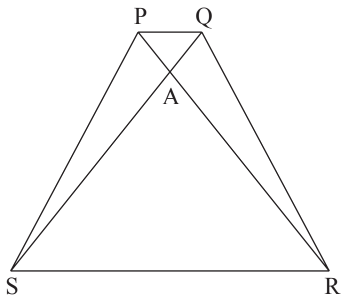
$$= \frac{\sec\theta + \tan\theta}{1} \qquad \ldots \left[\begin{array}{l} \because 1 + \tan^2\theta = \sec^2\theta \\ \therefore \sec^2\theta - \tan^2\theta = 1 \end{array}\right]$$

$= \text{RHS}$

$\therefore \text{LHS} = \text{RHS}$

$$\therefore \frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta.$$

Q. 3. (A) (i)



Activity :

In $\triangle$PQA and $\triangle$RSA,

$\angle$PQA $\cong$ $\angle$RSA ... **(Alternate interior angles)**

$\angle$PAQ $\cong$ $\angle$RAS ... **(Vertically opposite angles)**

$\therefore$ $\triangle$PQA $\sim$ $\triangle$RSA ... **AA test of similarity**

$\frac{PQ}{RS} = \frac{\boxed{\mathbf{AP}}}{AR}$ ... (Corresponding sides of similar triangles) ... (1)

Substituting AR = 5AP in (1)

$\therefore \frac{PQ}{RS} = \frac{\boxed{\mathbf{AP}}}{5AP}$

$\therefore \frac{PQ}{RS} = \frac{\boxed{\mathbf{1}}}{5}$

$\therefore$ RS = 5PQ.

(ii) Activity :

P (2, −2), Q (7, 3), R (11, −1) and S (6, −6) are the given points.

Using distance formula,

distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

PQ = $\boxed{\mathbf{5\sqrt{2}}}$ ... (1)

QR = $\boxed{\mathbf{4\sqrt{2}}}$ ... (2)

SR = $\boxed{\mathbf{5\sqrt{2}}}$ ... (3)

PS = $\boxed{\mathbf{4\sqrt{2}}}$ ... (4)

$\therefore$ from (1), (2), (3) and (4)

PQ = $\boxed{\mathbf{SR}}$

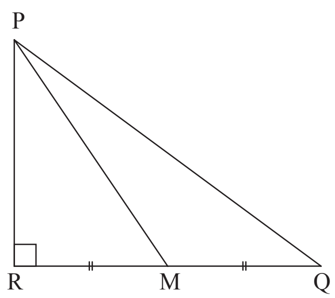
QR = $\boxed{\mathbf{PS}}$

$\therefore$ □PQRS is a parallelogram

... [A quadrilateral is a parallelogram, if its opposite sides are equal]

$\therefore$ P(2, −2), Q(7, 3), R(11, −1) and S(6, −6) are the vertices of a parallelogram.

Q. 3. (B) (i)



Proof : In △PRQ,

∠PRQ = 90° ... (Given)

$\therefore$ by Pythagoras theorem,

$PQ^2 = PR^2 + QR^2$ ... (1)

In $\triangle PRM$,

$\angle PRM = 90°$ ... (Given)

$\therefore$ by Pythagoras theorem,

$PM^2 = PR^2 + RM^2$ ... (2)

$RM = \frac{1}{2} RQ$ ... (M is the midpoint of seg RQ) ... (3)

$\therefore PM^2 = PR^2 + \left(\frac{1}{2} RQ\right)^2$ ... [From (2) and (3)]

$\therefore PM^2 = PR^2 + \frac{1}{4} RQ^2$

Multiplying each term with 4, we get

$4PM^2 = 4PR^2 + RQ^2$

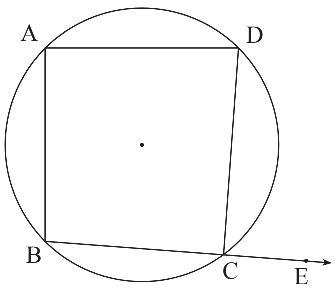
$\therefore 4PM^2 = 3PR^2 + (PR^2 + RQ^2)$

$\therefore 4PM^2 = 3PR^2 + PQ^2$ ... [From (1)]

$\therefore 4PM^2 - 3PR^2 = PQ^2$

i.e. $PQ^2 = 4PM^2 - 3PR^2$.

(ii)



Given : □ABCD is cyclic. $\angle DCE$ is an exterior angle of □ABCD.

To prove : $\angle DCE \cong \angle BAD$

Proof :

$\angle DCE + \angle BCD = 180°$ ... (Linear pair of angles) ... (1)

□ABCD is cyclic.

$\therefore \angle BAD + \angle BCD = 180°$ ... (Theorem of cyclic quadrilateral) ... (2)

$\therefore$ from (1) and (2), we get

$\angle DCE + \angle BCD = \angle BAD + \angle BCD$

$\therefore \angle DCE = \angle BAD$

$\therefore \angle DCE \cong \angle BAD$.

(iii) Solution :

Let seg AB and seg CD represent the two buildings. Seg BD represents the road.

Seg AE $\perp$ seg CD ... C–E–D.

$\angle$CAE is the angle of elevation.

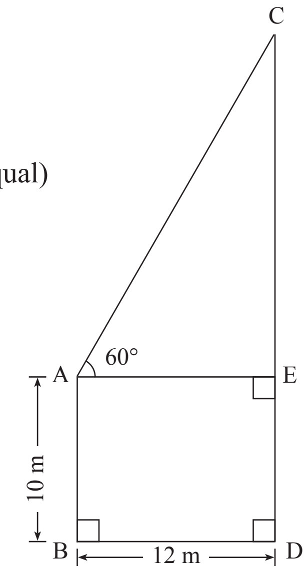
AB = 10 m, BD = 12 m and $\angle$CAE = 60°.

$\square$ABDE is a rectangle

DE = AB = 10 m

AE = BD = 12 m ... (Opposite sides of rectangle are equal)

In right angled $\triangle$CEA,



$\tan 60° = \frac{CE}{AE}$ ... (By definition)

$\therefore \sqrt{3} = \frac{CE}{12}$ ... ($\tan 60° = \sqrt{3}$)

$\therefore$ CE = $12\sqrt{3}$ m

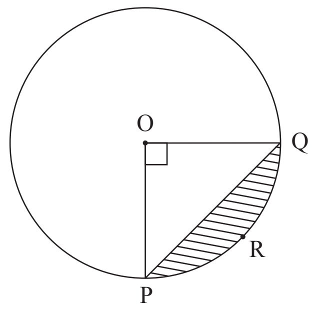
CD = CE + ED ... (C–E–D)

$\therefore$ CD = $(12\sqrt{3} + 10)$ m

Ans. The height of second building is **$(12\sqrt{3} + 10)$ m.**

(iv) Solution : Area of the shaded region is 114 cm^2.

The shaded region is segment PRQ.



A(segment PRQ) = 114 cm^2

m(arc PRQ) = $\angle$POQ = θ = 90°

$$A(\text{segment PRQ}) = r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$$

$$\therefore 114 = r^2\left[\frac{3.14 \times 90}{360} - \frac{\sin 90°}{2}\right]$$

$$\therefore 114 = r^2\left[\frac{3.14}{4} - \frac{1}{2}\right] \quad \text{... } (\sin 90° = 1)$$

$$\therefore 114 = r^2\left[\frac{3.14 - 2}{4}\right]$$

$$\therefore 114 = r^2 \times \frac{1.14}{4}$$

$$\therefore r^2 = \frac{114 \times 4 \times 100}{1.14 \times 100}$$

$$\therefore r^2 = \frac{114 \times 4 \times 100}{114}$$

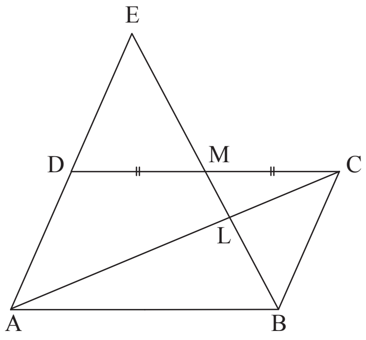
$\therefore r^2 = 400$

$\therefore r = \sqrt{400}$

$\therefore r = 20$ cm

Ans. The radius of the circle is **20 cm.**

Q. 4. (i)



Proof : Side AD $\parallel$ side BC ... (Opposite sides of a parallelogram)

i.e. AE $\parallel$ BC and BE is the transversal.

$\therefore \angle BEA \cong \angle EBC$ ... (Alternate angles)

i.e. $\angle MED \cong \angle MBC$

$\angle DME \cong \angle BMC$ ... (Vertically opposite angles)

$\therefore \triangle EMD \sim \triangle BMC$ ... (AA test of similarity)

$\therefore \frac{MD}{MC} = \frac{ED}{BC}$ ... (Corresponding sides of similar triangles are proportional)

$MD = MC$ ... (M is the midpoint of DC)

$\therefore \frac{MD}{MC} = \frac{ED}{BC} = 1$

$\therefore ED = BC$ ... (1)

$BC = AD$ ... (Opposite sides of a parallelogram) ... (2)

From (1) and (2), $ED = AD$

$AD + ED = AD + AD$ ... (Adding AD to both the sides)

$\therefore AE = 2AD$ ... (A – D – E) ... (3)

$\therefore AE = 2BC$ ... ($\because$ AD = BC, opposite sides of parallelogram)

AE $\parallel$ BC and AC is the transversal.

$\therefore \angle EAC \cong \angle BCA$ ... (Alternate angles)

$\angle ELA \cong \angle BLC$ ... (Vertically opposite angles)

$\therefore \triangle AEL \sim \triangle CBL$ ... (AA test of similarity)

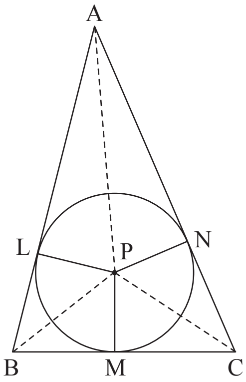
$\therefore \frac{EL}{BL} = \frac{AE}{BC}$ ... (Corresponding sides of similar triangles are proportional)

$\therefore \frac{EL}{BL} = \frac{2BC}{BC}$ ... [From (3)]

$\therefore \frac{EL}{BL} = 2$

$\therefore EL = 2BL.$

(ii)



Proof : Draw seg PA,
seg PL, seg PN, seg PB,
seg PC and seg PM

$PL = PM = PN = r$ ... (Radii of the same circle)

$PM \perp BC, PL \perp AB, PN \perp AC,$ ... (Tangent theorem)

$\therefore A(\triangle PBC) = \frac{1}{2} \times BC \times PM$

$A(\triangle PBC) = \frac{1}{2} \times BC \times r$ ... (1)

$A(\triangle PAC) = \frac{1}{2} \times AC \times r$ ... (2)

$A(\triangle PAB) = \frac{1}{2} \times AB \times r$ ... (3)

$\therefore A(\triangle ABC) = A(\triangle PBC) + A(\triangle PAC) + A(\triangle PAB)$ ... [From (1), (2) and (3)]

$= \frac{1}{2} \times BC \times r + \frac{1}{2} \times AC \times r + \frac{1}{2} \times AB \times r$

$= \frac{1}{2}(BC + AC + AB) \times r$

$A(\triangle ABC) = \frac{1}{2}(AB + BC + AC) \times r.$

(iii) For metallic cuboid,

length $(l) = 44$ cm

breadth $(b) = 42$ cm

height $(h) = 21$ cm

For the sphere,

Let the radius be r.

Volume of the sphere = Volume of metallic cuboid

$\frac{4}{3}\pi r^3 = l \times b \times h$

$\therefore \frac{4}{3} \times \frac{22}{7} \times r^3 = 44 \times 42 \times 21$

$\therefore r^3 = \frac{44 \times 42 \times 21 \times 3 \times 7}{4 \times 22}$

$\therefore\ r^3 = 9261$

$\therefore\ r = 21$ cm

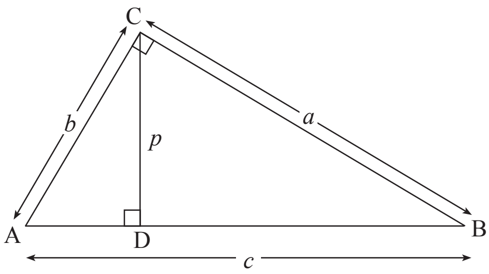
Surface area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 21 \times 21$$

$$= 5544 \text{ cm}^2$$

Ans. Surface area of the sphere is **5544 cm²**.

Q. 5. (i)



Proof : $\triangle DAC \sim \triangle CAB$ … (Similarity of right angled triangles)

$\therefore\ \frac{CD}{BC} = \frac{AC}{AB}$ … (c.s.s.t)

$\therefore\ \frac{p}{a} = \frac{b}{c}$ $\quad\therefore\ p = \frac{ab}{c}$

$\therefore\ p^2 = \frac{a^2b^2}{c^2}$ … (Squaring both the sides)

$\therefore\ \frac{1}{p^2} = \frac{c^2}{a^2b^2}$ … (By invertendo) … (1)

In right angled $\triangle CAB$, by Pythagoras theorem,

$AB^2 = BC^2 + AC^2$

$\therefore\ c^2 = a^2 + b^2$ … (2)

From (1) and (2),

$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2}$$

$$= \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$$

$$\therefore\ \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

i.e. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

(ii) Analysis :

For $\triangle ABC$, the lengths of three sides are known.

$\therefore$ $\triangle ABC$ can be constructed.

$\triangle ABC \sim \triangle LMN$

$\therefore \dfrac{AB}{LM} = \dfrac{BC}{MN} = \dfrac{AC}{LN}$ ... (Corresponding sides of similar triangles are in proportion)

$\therefore \dfrac{5.5}{LM} = \dfrac{6}{MN} = \dfrac{4.5}{LN} = \dfrac{5}{4}$

$\therefore \dfrac{5.5}{LM} = \dfrac{5}{4}$

$\therefore LM = \dfrac{5.5 \times 4}{5}$

$\therefore LM = 1.1 \times 4$

$\therefore LM = 4.4$ cm

$\dfrac{6}{MN} = \dfrac{5}{4}$

$\therefore MN = \dfrac{6 \times 4}{5}$

$\therefore MN = \dfrac{24}{5}$

$\therefore MN = 4.8$ cm

$\dfrac{4.5}{LN} = \dfrac{5}{4}$

$\therefore LN = \dfrac{4.5 \times 4}{5}$

$\therefore LN = \dfrac{18}{5}$

$\therefore LN = 3.6$ cm

$\therefore$ $\triangle LMN$ can be constructed.

Ans.