MATHEMATICS (PART – II)

SOLUTION : PRACTICE QUESTION PAPER 3

Q. 1. (A) (i) (C)

(ii) (D)

(iii) (B)

(iv) (B)

Q. 1. (A) Explanation to the answers to MCQs in this question has been given below for students' guidance. Please note that, **Students are not expected to write the explanation in the examination**.

Explanations :

(i) (C) [Ratio of areas of two similar triangle = ratio of square of their corresponding sides.]

$$\therefore \frac{A_1}{A_2} = \frac{(S_1)^2}{(S_2)^2}$$

(ii) (D) [Side of square $= \frac{1}{\sqrt{2}} \times$ diagonal, perimeter $= 4 \times$ side]

(iii) (B) Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(iv) (B) $\left[\begin{array}{ll} C = 2\pi r, & \\ A = \pi r^2 & \\ \therefore \frac{2\pi r}{\pi r^2} = \frac{2}{7} & \therefore r = 7 \\ & \therefore d = 14 \end{array} \right]$

Q. 1. (B) **(i) Solution :** Volume of the cylinder = 600 cm^3 ... (Given)

The cylinder and cone have equal base radii and perpendicular heights.

$$\therefore \text{ Volume of the cone} = \frac{1}{3} \times \text{Volume of cylinder}$$

$$= \frac{1}{3} \times 600$$

$$= 200 \text{ cm}^3$$

Ans. Volume of the cone is **200 cm^3**.

(ii) Solution : Slope of X-axis is 0

Parallel lines have equal slopes.

Ans. Thus slope of line parallel to X-axis is **0**.

(iii) Solution : In $\triangle QPR$,

$$\left.\begin{aligned} \angle QPR &= 90^\circ \\ \angle PQR &= 60^\circ \\ \angle PRQ &= 30^\circ \end{aligned}\right\} \quad \text{... (Given)}$$

$\therefore$ $\triangle QPR$ is a $30^\circ - 60^\circ - 90^\circ$ triangle

$\therefore$ By $30^\circ - 60^\circ - 90^\circ$ triangle theorem,

$PQ = \frac{1}{2} QR$ ... (Side opposite to 30°)

$\therefore$ $7 = \frac{1}{2} QR$

$\therefore$ $QR = 7 \times 2$

$\therefore$ $QR = 14$ cm

Ans. $QR =$ **14 cm.**

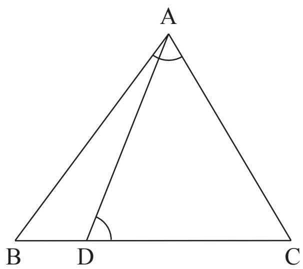
(iv) Solution : $\frac{AX}{XB} = \frac{2}{4} = \frac{1}{2}$ and $\frac{AY}{YC} = \frac{3}{6} = \frac{1}{2}$

$\therefore$ $\frac{AX}{XB} = \frac{AY}{YC}$

$\therefore$ line $XY \parallel$ side BC ... (By converse of basic proportionality theorem)

Ans. By converse of basic proportionality theorem, line XY $\parallel$ side BC.

Q. 2. (A) (i) Activity :



In $\triangle BAC$ and $\triangle ADC$,

$\angle BAC \cong \angle ADC$ ... (Given)

$\angle ACB \cong$ **$\boxed{\angle DCA}$** ... (Common angle)

$\therefore$ $\triangle BAC \sim \triangle ADC$ ... (**$\boxed{AA}$** test of similarity)

$\therefore$ $\frac{CA}{\boxed{CD}} = \frac{CB}{\boxed{CA}}$ ... (Corresponding sides of similar triangles)

$\therefore$ $CA^2 = CB \times CD$.

(ii) Activity :

$$1+\cot^2\theta=\operatorname{cosec}^2\theta$$

$$1+\boxed{\left(\frac{\mathbf{40}}{\mathbf{9}}\right)}^2=\operatorname{cosec}^2\theta \qquad \text{... (Substituting the values)}$$

$$\therefore\ 1+\frac{1600}{81}=\operatorname{cosec}^2\theta$$

$$\therefore\ \operatorname{cosec}^2\theta=\boxed{\frac{\mathbf{1681}}{\mathbf{81}}}$$

$$\therefore\ \operatorname{cosec}\theta=\boxed{\frac{\mathbf{41}}{\mathbf{9}}}$$

$$\therefore\ \sin\theta=\frac{1}{\operatorname{cosec}\theta}$$

$$\therefore\ \sin\theta=\boxed{\frac{\mathbf{9}}{\mathbf{41}}}.$$

(iii) Activity :

For cylindrical bucket :	**For sand cone :**
diameter = 28 cm	height (h_1) = 14 cm
radius (r) = 14 cm	Let the radius be r_1
height (h) = 20 cm	base area = πr_1^2

Here, volume of cylindrical bucket = Volume of the sand cone

$$\pi r^2 h=\frac{1}{3}\pi r_1^2 h_1$$

$$\therefore\ \frac{22}{7}\times 14\times 14\times\boxed{\mathbf{20}}=\frac{1}{3}\pi r_1^2\times\boxed{\mathbf{14}}$$

$$\therefore\ \pi r_1^2=\frac{22\times 14\times 14\times\boxed{\mathbf{20}}\times 3}{7\times 14}$$

$$\therefore\ \pi r_1^2=\text{base area of cone}=\boxed{\mathbf{2640}}\ \text{cm}^2.$$

Q. 2. (B) (i) Solution :

Let radius of first circle is r_1 and radius of second circle is r_2.

Both the circles touching internally.

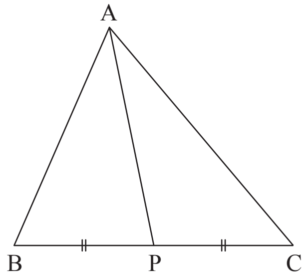
$\therefore\ |r_1-r_2|=10$

$\therefore\ r_1-r_2=10$ ... (1) or $r_1-r_2=-10$ ... (2)

$\therefore\ 15-r_2=10$ ... [From (1)]	$15-r_2=-10$
$\therefore\ 15-10=r_2$	$\therefore\ 15+10=r_2$
$\therefore\ r_2=5$ cm	$\therefore\ r_2=25$ cm

Ans. Radius and second circle is **5 cm** or **25 cm**.

(ii)



BC = 18 ... (Given)

$BP = \frac{1}{2} BC$ ... (P is the midpoint of seg BC)

$\therefore BP = \frac{1}{2} \times 18$

$\therefore BP = 9$

In $\triangle ABC$,

seg AP is the median ... (Given)

$\therefore$ by Apollonius theorem,

$AB^2 + AC^2 = 2AP^2 + 2BP^2$

$\therefore 260 = 2AP^2 + 2(9)^2$

$\therefore 260 = 2AP^2 + 162$

$\therefore 2AP^2 = 260 - 162$

$\therefore 2AP^2 = 98$

$\therefore AP^2 = \frac{98}{2}$

$\therefore AP^2 = 49$

$\therefore AP = 7$ ... (Taking square roots of both the sides)

Ans. AP = **7.**

(iii) Solution : Let A(− 7, 6), B(2, − 2) and C(8, 5).

Let $A(-7, 6) \equiv (x_1, y_1)$

$B(2, -2) \equiv (x_2, y_2)$

$C(8, 5) \equiv (x_3, y_3)$

Let $G(x, y)$ be the centroid.

By centroid formula,

$x = \frac{x_1 + x_2 + x_3}{3}$

$\therefore x = \frac{-7 + 2 + 8}{3}$

$\therefore x = \frac{3}{3}$

$\therefore x = 1$

$y = \frac{y_1 + y_2 + y_3}{3}$

$\therefore y = \frac{6 + (-2) + 5}{3}$

$\therefore y = \frac{9}{3}$

$\therefore y = 3$

Ans. The coordinates of the centroid are **(1, 3).**

(iv) Proof :

$$\text{LHS} = \frac{\sin^2\theta}{\cos\theta} + \cos\theta$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \sec\theta$$

$$= \text{RHS}$$

$$\therefore \frac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta.$$

(v) Solution : Diameter of a sphere = 6 cm

its radius $(r) = 3$ cm

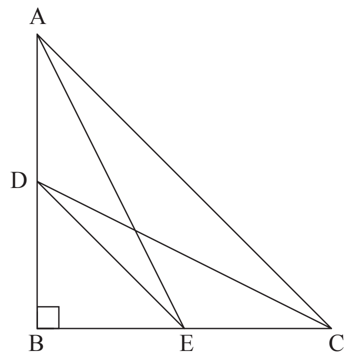
$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.14 \times 3 \times 3 \times 3$$

$$= 113.04 \text{ cm}^3$$

Ans. Volume of the sphere is **113.04 cm³.**

Q. 3. (A) (i) Activity :



In right angled $\triangle ABE$,

by Pythagoras theorem,

$AE^2 = AB^2 + \boxed{\mathbf{BE^2}}$ ... (1)

In right angled $\triangle DBC$,

by Pythagoras theorem,

$CD^2 = \boxed{\mathbf{DB^2}} + BC^2$ ... (2)

In right angled $\triangle ABC$,

by Pythagoras theorem,

$AC^2 = AB^2 + \boxed{\mathbf{BC^2}}$ ... (3)

In right angled $\triangle DBE$,

by Pythagoras theorem,

$DE^2 = BD^2 + \boxed{\mathbf{BE^2}}$ ... (4)

Adding (1) and (2), we get

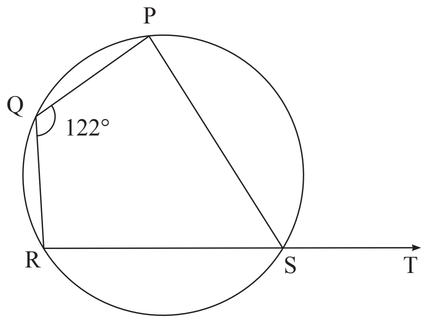
$AE^2 + CD^2 = AB^2 + BE^2 + BD^2 + BC^2$

$\therefore AE^2 + CD^2 = (AB^2 + BC^2) + (BD^2 + BE^2)$ ... (5)

Substituting the values of (3) and (4) in (5), we get

$AE^2 + CD^2 = \boxed{\mathbf{AC^2}} + \boxed{\mathbf{DE^2}}$.

(ii)



Activity :

□ PQRS is cyclic.

Opposite angles of a cyclic quadrilateral are **supplementary**

$\angle PQR = 122°$ ... (Given)

and $\angle PQR + \angle PSR =$ **180°**

$\therefore$ $122° + \angle PSR =$ **180°**

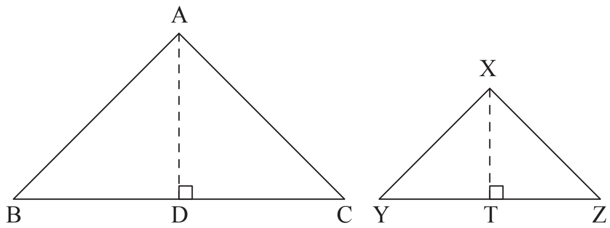
$\therefore$ $\angle PSR =$ **58°**

$\therefore$ $\angle PST$ is an exterior angle of cyclic quadrilateral PQRS.

$\angle PST = \angle$ **PQR** ... (Corollary of cyclic quadrilateral theorem)

$\therefore$ $\angle PST =$ **122°**.

Q. 3. (B) (i)



Given : $\triangle ABC \sim \triangle XYZ$

To prove : $\dfrac{A(\triangle ABC)}{A(\triangle XYZ)} = \dfrac{AB^2}{XY^2} = \dfrac{BC^2}{YZ^2} = \dfrac{AC^2}{XZ^2}$

Construction : Draw seg AD $\perp$ side BC such that B – D – C and seg XT $\perp$ side YZ such that Y – T – Z.

Proof : Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle XYZ)} = \dfrac{BC \times AD}{YZ \times XT}$

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle XYZ)} = \dfrac{BC}{YZ} \times \dfrac{AD}{XT}$ ... (1)

$\triangle ABC \sim \triangle XYZ$ ... (Given)

$\therefore \angle B \cong \angle Y$ ... (Corresponding angles of similar triangles) ... (2)

and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$ ... (Corresponding sides of similar triangles are in proportion) ... (3)

In $\triangle$ABD and $\triangle$XYT,

$\angle ABD \cong \angle XYT$ ... [From (2)]

$\angle ADB \cong \angle XTY$ ... (Each measures 90°)

$\therefore \triangle ABD \sim \triangle XYT$ ... (AA test of similarity)

$\therefore \frac{AB}{XY} = \frac{AD}{XT}$ ... (Corresponding sides of similar triangles are in proportion) ... (4)

$\therefore$ from (3) and (4), we get

$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AD}{XT}$.

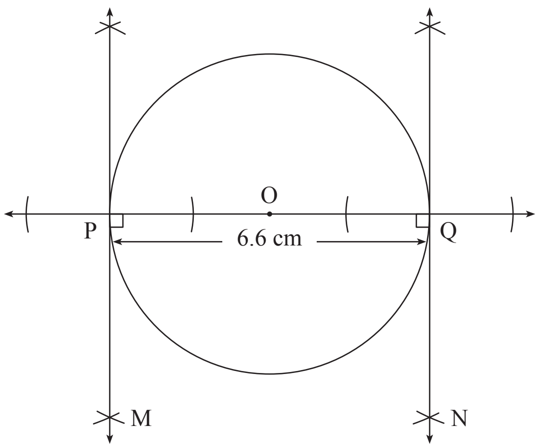
$\therefore \frac{BC}{YZ} = \frac{AD}{XT}$ ... (5)

$\frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{BC}{YZ} \times \frac{BC}{YZ}$ ... [From (1) and (5)]

$\therefore \frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{BC^2}{YZ^2}$ ... (6)

$\therefore \frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{AB^2}{XY^2} = \frac{BC^2}{YZ^2} = \frac{AC^2}{XZ^2}$ ... [From (1) and (6)]

(ii)



The tangents drawn at points P and Q are parallel.

(iii) Solution : A(− 1, 7) and B(4, − 3).

Let the coordinates of point P be (x, y).

Suppose coordinates of point A are (x_1, y_1) and B are (x_2, y_2) then $x_1 = -1$, $y_1 = 7$, $x_2 = 4$ and $y_2 = -3$.

P divides AB in the ratio 2 : 3.

$\therefore m : n = 2 : 3$

By section formula,

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\therefore\ x = \frac{2(4) + 3(-1)}{2 + 3}$$

$$\therefore\ x = \frac{8 - 3}{5}$$

$$\therefore\ x = \frac{5}{5}$$

$$\therefore\ x = 1$$

$$y = \frac{my_2 + ny_1}{m + n}$$

$$\therefore\ y = \frac{2(-3) + 3(7)}{2 + 3}$$

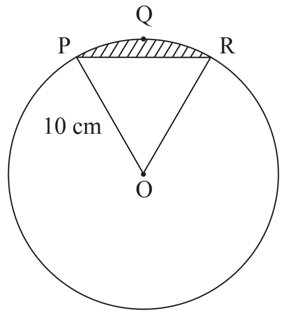
$$\therefore\ y = \frac{-6 + 21}{5}$$

$$\therefore\ y = \frac{15}{5}$$

$$\therefore\ y = 3$$

Ans. P(1, 3) divides the line segment joining the points A$(-1, 7)$ and B$(4, -3)$ in the ratio 2 : 3.

(iv) Solution :



The radius (r) of the circle = OP = 10 cm.

m(arc PQR) = θ = 60°.

The shaded region is segment PQR,

$$A(\text{segment PQR}) = r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$$

$$= 10^2\left[\frac{3.14 \times 60}{360} - \frac{\sin\theta}{2}\right]$$

$$= 100\left[\frac{3.14}{6} - \frac{\sqrt{3}}{2 \times 2}\right] \qquad \ldots \left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$$

$$= 100 \times \left[\frac{3.14}{6} - \frac{1.73}{4}\right] \qquad \ldots (\because \sqrt{3} = 1.73)$$

$$= 100\left[\frac{3.14 \times 2 - 1.73 \times 3}{12}\right]$$

$$= 100\left[\frac{6.28 - 5.19}{12}\right]$$

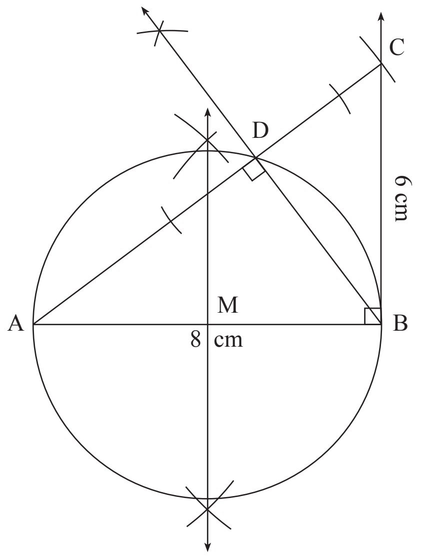
$$= 100 \times \frac{1.09}{12}$$

$$= \frac{109}{12}$$

$$\approx 9.08 \text{ cm}^2$$

Ans. Area of the shaded region is **9.08 cm²**.

Q. 4. **(i)**



Proof : A circle passes through the points A, D and B, $\angle ADB = 90°$

So arc ADB is a semicircle.

Hence AB is the diameter.

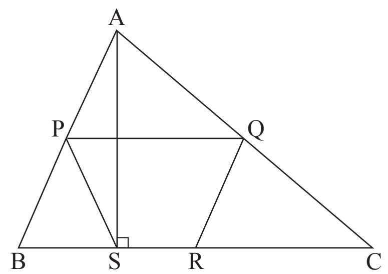
M is the midpoint of the diameter and hence the centre of the circle.

$\therefore$ MB is the radius.

CB is perpendicular to radius MB at its outer end B.

Thus CB is tangent to the circle … (Converse of tangent theorem)

(ii)



Proof :

In $\triangle ABC$, P and Q are the midpoints of side AB and side AC respectively.

$\therefore$ PQ $\parallel$ BC ... (Midpoint theorem)

$\therefore$ PQ $\parallel$ BR ... (B – R – C) ... (1)

Similarly, QR $\parallel$ BP ... (2)

$\therefore$ from (1) and (2) $\square$ PQRB is a parallelogram.

$\angle B = \angle PQR$ ... (Opposite angles of parallelogram are equal) ... (3)

In $\triangle$ ASB,

$\angle ASB = 90°$ ... (Given)

Seg SP is median drawn to hypotenuse AB.

$SP = \frac{1}{2} AB$ ... $\left[\begin{array}{c}\text{In a right angled triangle, the median drawn} \\ \text{to the hypotenuse is half the hypotenuse}\end{array}\right]$ ... (4)

also $PB = \frac{1}{2} AB$ ... (P is the midpoint of AB) ... (5)

In $\triangle PBS$,

$SP = PB$ ... [From (4) and (5)]

$\therefore \angle B = \angle PSB$ ... (Isosceles triangle theorem) ... (6)

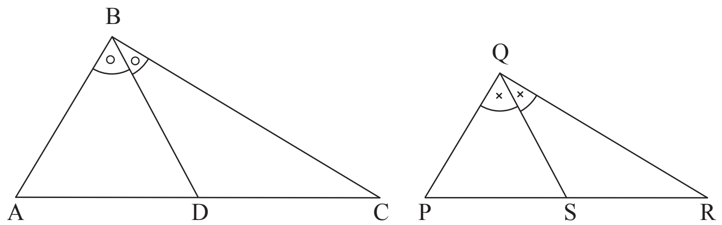
$\therefore \angle PSB = \angle PQR$ ... [From (3) and (6)] ... (7)

$\angle PSB + \angle PSR = 180°$ ... (Linear pair of angles)

$\therefore \angle PQR + \angle PSR = 180°$ ... [From (7)]

$\therefore \square PQRS$ is cyclic ... [A quadrilateral is cyclic, if its opposite angles are supplementary]

(iii)



Proof :

$\frac{AD}{PS} = \frac{DC}{SR}$ … (Given)

$\therefore$ by alternendo,

$\frac{AD}{DC} = \frac{PS}{SR}$ … (1)

In $\triangle ABC$,

BD bisects $\angle ABC$

$\frac{AB}{BC} = \frac{AD}{DC}$ … (2) [Theorem of angle bisector of a triangle]

In $\triangle PQR$,

QS bisects $\angle PQR$

$\therefore \frac{PQ}{QR} = \frac{PS}{SR}$ … (3) [Theorem of angle bisector of a triangle]

$\therefore$ from (1), (2) and (3), we get

$\frac{AB}{BC} = \frac{PQ}{QR}$

$\therefore$ by alternendo, we get

$\frac{AB}{PQ} = \frac{BC}{QR}$ … (4)

In $\triangle ABC$ and $\triangle PQR$,

$\frac{AB}{PQ} = \frac{BC}{QR}$ … [From (4)]

$\angle ABC \cong \angle PQR$ … (Given)

$\therefore \triangle ABC \sim \triangle PQR$ (SAS test of similarity)

Q. 5. (i) Solution :

$\tan\theta + \frac{1}{\tan\theta} = 2$ … (Given)

Squaring both the sides, we get

$$\left(\tan\theta + \frac{1}{\tan\theta}\right)^2 = 2^2$$

$$\tan^2\theta + 2 \times \tan\theta \times \frac{1}{\tan\theta} + \frac{1}{\tan^2\theta} = 4$$

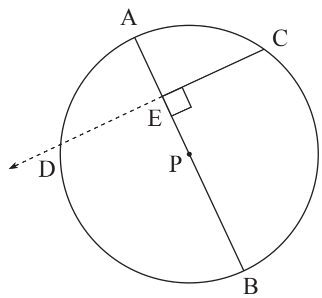
$$\tan^2\theta + 2 + \frac{1}{\tan^2\theta} = 4$$

$$\tan^2\theta + \frac{1}{\tan^2\theta} = 4 - 2$$

$$\tan^2\theta + \frac{1}{\tan^2\theta} = 2$$

Ans. The value of $\tan^2\theta + \frac{1}{\tan^2\theta}$ is **2**.

(ii) (a)



(b) Seg PE ⊥ chord CD.

Perpendicular drawn from the centre of the circle to the chord bisects the chord

$\therefore$ CE = ED … (i)

(c) Chords AB and CD intersect at point E inside the circle

$\therefore$ CE × ED = AE × EB … (ii)

(d) CE × ED = AE × EB … [From (ii)]

$\therefore$ CE × CE = AE × EB … [From (i)]

$\therefore CE^2 = AE \times EB$

$\therefore$ CE is the geometric mean of AE and EB.