MATHEMATICS (PART–II)

SOLUTION : PRACTICE QUESTION PAPER 2

Q. 1. (A) (i) (C)

(ii) (A)

(iii) (B)

(iv) (A)

Q. 1. (A) Explanation to the answers to MCQs in this question has been given below for students' guidance. Please note that, **Students are not expected to write the explanation in the examination**.

Explanations :

(i) (C) [In a triplet, if the square of largest number is equal to the sum of the squares of the remaining two numbers, then the group of three number is called Pythagorean triplet.]

(ii) (A) [If chord AB and CD intersect internally, at point E then AE $\times$ EB = CE $\times$ ED … (Theorem of internal division of chords)]

(iii) (B) $\left[\text{If } \triangle \text{DEF} \sim \triangle \text{QRP then } \frac{\text{DE}}{\text{QR}} = \frac{\text{EF}}{\text{RP}} = \frac{\text{DF}}{\text{QP}}\right]$

(iv) (A) [On X-axis y-coordinate of each point is zero and x-coordinate on left side of origin is negative.]

Q. 1. (B) **(i) Solution :** radius of the cone $(r) = 7$ cm

Its perpendicular height $(h) = 24$ cm

$l^2 = r^2 + h^2$

$= 7^2 \times 24^2$

$= 49 + 576$

$= 625$

$\therefore\ l = 25$ cm

Ans. The slant height of the cone is **25 cm**.

(ii) Solution : $\angle A + \angle C = 180°$ … (Opposite angles of cyclic quadrilateral)

$\therefore\ 80° + \angle C = 180°$

$\therefore\ \angle C = 180° - 80° = 100°$

Ans. Measure of $\angle C$ is **100°**.

(iii) Solution : In $\triangle$ABC,

$\angle ABC = 90°$ … (Given)

$\therefore$ by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore\ AC^2 = 324$

$\therefore AC = 18$ cm

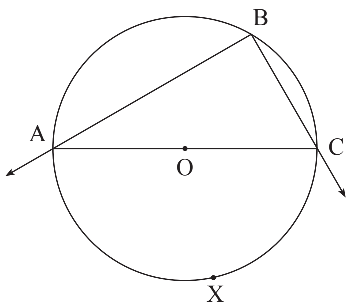
Ans. Length of AC is **18 cm**.

(iv) Solution : Inclination of the line $(\theta) = 60°$

Slope of the line $= \tan\theta = \tan 60° = \sqrt{3}$.

Ans. Slope of given line is $\sqrt{\mathbf{3}}$.

Q. 2. (A) (i)



Activity :

$m(\text{arc AXC}) = \boxed{\mathbf{180°}}$ ... (Measure of a semicircle)

$\angle ABC = \frac{1}{2}\boxed{\boldsymbol{m}\textbf{(arc AXC)}}$ ... (Inscribed angle theorem)

$\therefore\ \angle ABC = \frac{1}{2} \times \boxed{\mathbf{180°}}$

$\therefore\ \angle ABC = \boxed{\mathbf{90°}}$.

(ii) Activity :

$\sin^2\theta + \cos^2\theta = \boxed{\mathbf{1}}$

Dividing each term by $\cos^2\theta$, we get

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{\boxed{\mathbf{1}}}{\cos^2\theta}$$

$\therefore\ \boxed{\mathbf{tan^2}\,\theta} + 1 = \boxed{\mathbf{sec^2}\,\theta}$.

(iii) Activity :

Here $r_1 = 14$ cm, $r_2 = 7$ cm, $h = 30$ cm

Volume of bucket $= \frac{1}{3}\pi(r_1{}^2 + r_2{}^2 + r_1 + r_2) \times h$

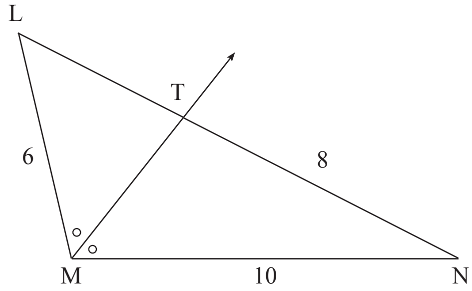
$= \frac{1}{3} \times \frac{22}{7} \times \boxed{\mathbf{14^2 + 7^2 + 14 \times 7}} \times 30$ ... (Substituting the values)

$= \frac{22}{7} \times \boxed{\mathbf{343}} \times 10$

$= \boxed{\textbf{10780}}\ \text{cm}^3$

$= \boxed{\textbf{10.78}}\ \text{litres} \quad \ldots [1 \text{ litre} = 1000 \text{ cm}^3]$

Q. 2. (B) (i) Solution :



In $\triangle$LMN,

ray MT bisects $\angle$LMN

$\therefore$ by the theorem of an angle bisector of a triangle.

$$\frac{LM}{MN} = \frac{LT}{TN}$$

$$\therefore \frac{6}{10} = \frac{LT}{8}$$

$$\therefore LT = \frac{6 \times 8}{10}$$

$\therefore$ LT $= 4.8$

Ans. LT $=$ **4.8**

(ii) Solution : $PQ^2 = (\sqrt{8})^2 = 8$

$QR^2 = (\sqrt{5})^2 = 5$

$PR^2 = (\sqrt{3})^2 = 3$

$QR^2 + PR^2 = 5 + 3 = 8$

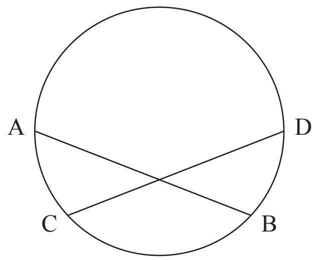
$\therefore QR^2 + PR^2 = PQ^2$

$\therefore$ by converse of Pythagoras theorem,

$\triangle$PQR is a right angled triangle. Here PQ is the hypotenuse. The angle opposite to the hypotenuse is the right angle.

Ans. $\triangle$PQR is a right angled triangle and $\angle PRQ = 90°$.

(iii)



Proof : Chord AB $\cong$ chord CD ... (Given)

$\therefore$ Arc ACB $\cong$ arc CBD ... (Arcs corresponding to congruent chords)

$\therefore$ m(arc ACB) = m(arc CBD) ... (1)

But m(arc ACB) = m(arc AC) + m(arc CB) ... (2)

and m(arc CBD) = m(arc CB) + m(arc BD) ... (3)

From (1), (2) and (3), we get

m(arc AC) + m(arc CB) = m(arc CB) + m(arc BD)

$\therefore$ m(arc AC) = m(arc BD)

$\therefore$ arc AC $\cong$ arc BD.

(iv) Solution : E(−4, −2) and F(6, 3)

$$\text{Slope of EF} = \frac{3-(-2)}{6-(-4)} = \frac{3+2}{6+4} = \frac{5}{10} = \frac{1}{2}$$

Ans. The slope of a line is $\mathbf{\frac{1}{2}}$.

(v) Proof :

$$\text{LHS} = \operatorname{cosec}\theta \sqrt{1-\cos^2\theta}$$

$$= \operatorname{cosec}\theta \times \sqrt{\sin^2\theta} \quad \ldots [\sin^2\theta + \cos^2\theta = 1, \therefore \sin^2\theta = 1-\cos^2\theta]$$

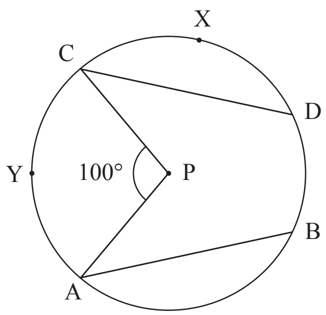
$$= \operatorname{cosec}\theta \times \sin\theta$$

$$= \frac{1}{\sin\theta} \times \sin\theta$$

$$= 1$$

$$= \text{RHS.}$$

Q. 3. (A) (i)



Activity :

$m(\text{arc AYC}) = \angle CPA$ ... (By definition of measure of minor arc)

$\therefore m(\text{arc AYC}) = \boxed{\mathbf{100°}}$

chord AB $\cong$ chord CD ... (Given)

arc AB $\cong$ $\boxed{\textbf{arc CXD}}$ ... (Corresponding minor arcs related to congruent chords)

$\therefore m(\text{arc AB}) = m(\text{arc CXD}) = 105°$

Now,

$m(\text{arc BD}) + m(\text{arc AB}) + \boxed{\mathbf{m(arc\ AYC)}} + m(\text{arc CXD}) = 360°$

... (Measure of a circle)

$\therefore m(\text{arc BD}) + 105° + \boxed{\mathbf{100°}} + 105° = 360°$

$\therefore m(\text{arc BD}) + \boxed{\mathbf{310°}} = 360°$

$\therefore m(\text{arc BD}) = \boxed{\mathbf{50°}}$.

(ii) Proof :

$$\text{LHS} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta\left(\boxed{\mathbf{(1-2\sin^2\theta)}}\right)}{\cos\theta\left(\boxed{\mathbf{(2\cos^2\theta - 1)}}\right)}$$

Replacing 1 with $\sin^2\theta + \cos^2\theta$

$$= \frac{\sin\theta\,(\sin^2\theta + \cos^2\theta - \boxed{\mathbf{2\sin^2\theta}})}{\cos\theta\,(2\cos^2\theta - \boxed{\mathbf{(\sin^2\theta + \cos^2\theta)}})} \quad \ldots (\because \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{\sin\theta\ \boxed{\mathbf{(\cos^2\theta - \sin^2\theta)}}}{\cos\theta\ (\cos^2\theta - \sin^2\theta)} = \frac{\sin\theta}{\boxed{\mathbf{\cos\theta}}}$$

$= \tan\theta$

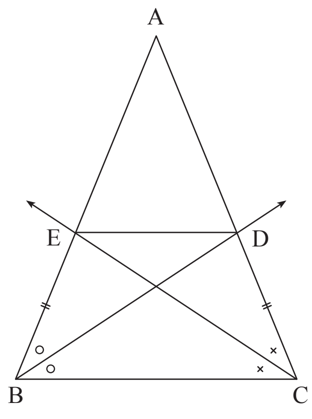
$= \text{RHS}.$

Q. 3. (B) (i) Proof : In $\triangle ABC$,

ray BD is the bisector of $\angle ABC$

$\therefore$ by the theorem of an angle bisector of a triangle,

$$\frac{AB}{BC} = \frac{AD}{DC} \quad \ldots (1)$$



In $\triangle ABC$,

ray CE is the bisector of $\angle ACB$

$\therefore$ by the theorem of an angle bisector of a triangle,

$$\frac{AC}{BC} = \frac{AE}{EB} \quad \ldots (2)$$

Seg AB $\cong$ seg AC ... (Given) ... (3)

$\therefore \frac{AB}{BC} = \frac{AC}{BC}$ ... [From (1), (2) and (3)] ... (4)

In $\triangle$ABC,

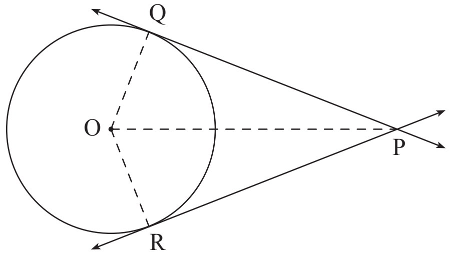
$\frac{AE}{EB} = \frac{AD}{DC}$ ... [From (1), (2) and (4)]

$\therefore$ by converse of basic proportionality theorem,

seg ED $\parallel$ side BC

i.e. ED $\parallel$ BC.

(ii)



Given : (1) A circle with centre O.

(2) Lines PQ and PR are tangents to the circle at points Q and R respectively.

To prove : seg PQ $\cong$ seg PR

Construction : Draw seg OP, seg OQ and seg OR

Proof : In $\triangle$OQP and $\triangle$ORP,

$\angle OQP = \angle ORP = 90°$ ... (Tangent theorem)

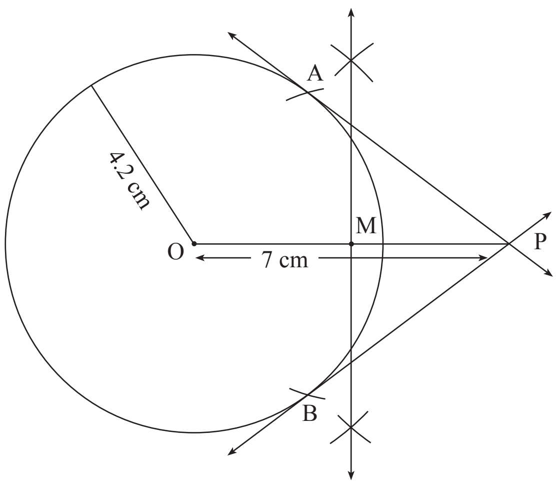
Hypotenuse OP $\cong$ Hypotenuse OP ... (Common side)

side OQ $\cong$ side OR ... (Radii of the same circle)

$\therefore \triangle OQP \cong \triangle ORP$ ... (Hypotenuse side test)

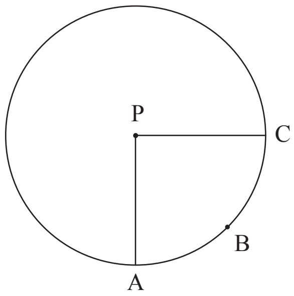
$\therefore$ seg PQ $\cong$ seg PR ... (c.s.c.t.)

(iii)



length of tangent segment PA and PB is 5.6 cm.

(iv) Solution :



The radius (r) of the circle = 14 cm.

$A(P-ABC) = 154 \text{ cm}^2$.

Let $m(\text{arc ABC}) = \angle APC = \theta$

$$A(P-ABC) = \frac{\theta}{360} \times \pi r^2$$

$$\therefore\ 154 = \frac{\theta}{360} \times \frac{22}{7} \times 14 \times 14$$

$$\therefore\ \theta = \frac{154 \times 360 \times 7}{22 \times 14 \times 14}$$

$\therefore\ \theta = 90°$ $\quad\therefore\ \angle APC = 90°$

$$\text{Area of the sector} = \frac{\text{length of the arc} \times \text{radius}}{2}$$

$$\therefore\ A(P-ABC) = \frac{l(\text{arc ABC}) \times r}{2}$$

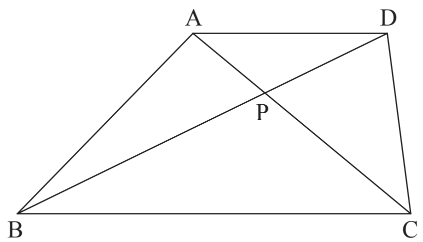
$$\therefore\ 154 = \frac{l(\text{arc ABC}) \times 14}{2}$$

$$\therefore\ l(\text{arc ABC}) = \frac{154}{7}$$

$\therefore\ l(\text{arc ABC}) = 22$ cm

Ans. $\angle APC =$ **90°** and $l(\text{arc ABC}) =$ **22 cm.**

Q. 4. **(i)**



Proof :

In $\triangle BPC$ and $\triangle DPA$,

$\angle CBP \cong \angle ADP$ … (Alternate angles)

$\angle BPC \cong \angle DPA$ … (Vertically opposite angles)

$\therefore\ \triangle BPC \sim \triangle DPA$ … (AA test of similarity)

$\therefore\ \dfrac{BP}{DP} = \dfrac{CP}{AP}$ … (Corresponding sides of similar triangles) … (1)

$AP = \frac{1}{3} AC$ ... (Given)

$\therefore\ 3AP = AC$

$\therefore\ 3AP = AP + CP$

$\therefore\ 3AP - AP = CP$

$\therefore\ 2AP = CP$

$\therefore\ \frac{AP}{CP} = \frac{1}{2}$

$\therefore\ \frac{CP}{AP} = \frac{2}{1}$ ... (By Invertendo) ... (2)

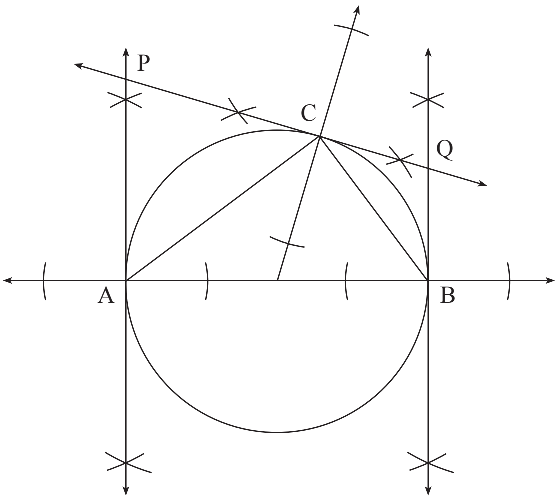
$\therefore$ from (1) and (2)

$\frac{BP}{DP} = \frac{2}{1}$

$\therefore\ 2DP = BP$

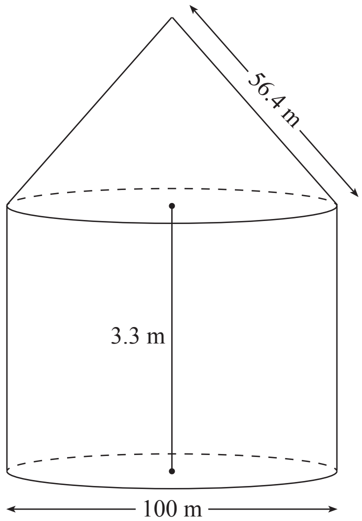
$\therefore\ DP = \frac{1}{2} BP.$

(ii) Ans.



$\square$ABQP is a **trapezium** formed due to intersection of tangents and the chord.

(iii) Solution :



For the cylindrical part of the tent :

Diameter $= 100$ m

$\therefore$ radius $(r) = \frac{1}{2} \times 100 = 50$ m

height $(h) = 3.3$ m

For the conical part of the tent :

radius $(r) = 50$ cm

slant height $(l) = 56.4$ m

Area of the canvas used to make the tent = Curved surface area of the cylindrical part of the tent + Curved surface area of the conical part of the tent

$= 2\pi rh + \pi rl$

$= \pi r(2h + l)$

$= \frac{22}{7} \times 50\ (2 \times 3.3 + 56.4)$

$= \frac{22}{7} \times 50 \times 63$

$= 22 \times 50 \times 9$

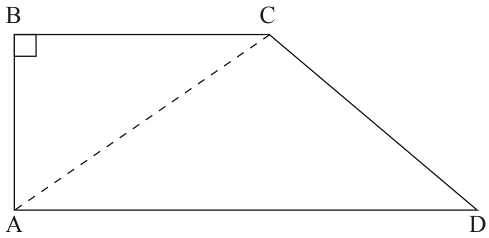
$= 9900\ \text{m}^2$

Cost of canvas per $\text{m}^2 =$ ₹ 8.

$\therefore$ total cost of canvas per $\text{m}^2 = 8 \times 9900 =$ ₹ 79,200.

Ans. Cost of canvas required for tent is **₹ 79,200**.

Q. 5. **(i)**



Solution :

(a) In $\triangle ABC$, $\angle ABC = 90°$

$\therefore$ by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$ … (1)

(b) $AD^2 = AB^2 + BC^2 + CD^2$ (Given) … (2)

Substituting (1) in (2), we get

$AD^2 = AC^2 + CD^2$ … (3)

(c) In $\triangle ACD$,

$AD^2 = AC^2 + CD^2$ … [From (3)]

$\therefore \triangle ACD$ is a right angled triangle and $\angle ACD = 90°$

... (By converse of Pythagoras theorem)

Ans. (a) $\mathbf{AC^2 = AB^2 + BC^2}$

(b) $\mathbf{AD^2 = AC^2 + CD^2}$

(c) $\angle$**ACD = 90°, converse of Pythagoras theorem.**

(ii) Solution :

A $(-2, -1)$, B $(p, 0)$, C $(4, q)$ and D $(1, 2)$ are the vertices of a parallelogram.

Diagonals of parallelogram bisect each other.

Coordinates of the midpoint of AC = Coordinates of the midpoint of BD

$$\left(\frac{-2+4}{2}, \frac{-1+q}{2}\right) = \left(\frac{p+1}{2}, \frac{0+2}{2}\right) \qquad \text{... (by midpoint formula),}$$

$$\therefore \left(1, \frac{-1+q}{2}\right) = \left(\frac{p+1}{2}, 1\right)$$

$\therefore \frac{p+1}{2} = 1$ and $\frac{-1+q}{2} = 1$

$\therefore p+1 = 2$ $\quad\therefore -1+q = 2$

$\therefore p = 2-1$ $\quad\therefore q = 2+1$

$\therefore p = 1$ $\quad\therefore q = 3$

Ans. Values of p and q are **1** and **3** respectively.