MATHEMATICS (PART – II)

SOLUTION : PRACTICE QUESTION PAPER 1

Q. 1. (A) (i) (B)

(ii) (C)

(iii) (A)

(iv) (D)

Q. 1. (A) Explanation to the answers to MCQs in this question has been given below for students' guidance. Please note that, **Students are not expected to write the explanation in the examination**.

Explanations :

(i) (B) $[(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2]$

(ii) (C) $\angle PAO = \angle PBO = 90°$ (Tangent-radius theorem)

$\angle APB = 70°$ $\therefore$ $\angle AOB = 110°$ (Remaining angle of quadrilateral)

$\angle POA = \frac{1}{2} \angle AOB = \frac{1}{2} \times 110° = 55°$

(iii) (A) Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(iv) (D) $\left[l = \frac{\theta}{360°} \times 2\pi r \quad \therefore\ 2\pi r = \text{Circumference of circle} = l \times \frac{360°}{\theta} \right]$

Q. 1. (B) **(i) Solution :** Side of a square = 8 cm

Diagonal of a square $= \sqrt{2} \times \text{side}$

$= \sqrt{2} \times 8$

$= 8\sqrt{2}$ cm

Ans. The diagonal of a square is $\mathbf{8\sqrt{2}}$ **cm**.

(ii) Solution : $\triangle ABC \sim \triangle LMN$

$\angle B = 60°$

$\angle M = \angle B = 60°$ … (Corresponding angles of similar triangles)

Ans. $\angle M = \mathbf{60°}$.

(iii) Solution : When two circles touch each other their point of contact lies on the line joining the centre.

When circles touch each other internally then distance between the centres is the difference of radii.

The required distance = 8 − 2 = 6 cm.

Ans. The distance between their centres is **6 cm**.

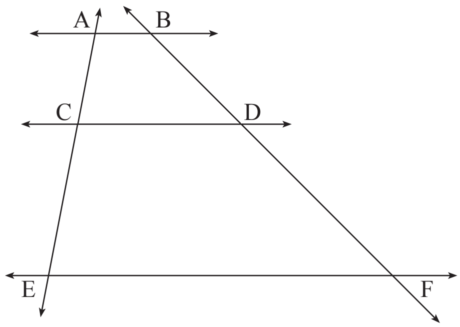
(iv) Solution : $\tan A = \sqrt{3}$ (Given)

as $\tan 60° = \sqrt{3}$, $\angle A = 60°$

$\sin A = \sin 60° = \dfrac{\sqrt{3}}{2}$

Ans. The value of sin A is $\mathbf{\dfrac{\sqrt{3}}{2}}$.

Q. 2. (A) (i) Activity :



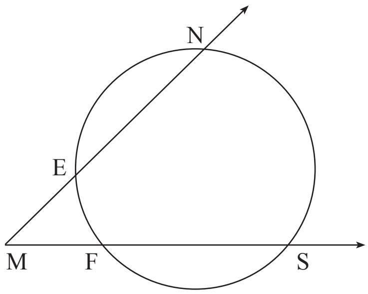
AB || CD || EF

$$\frac{AC}{\boxed{\mathbf{CE}}} = \frac{\boxed{\mathbf{BD}}}{DF}$$

$$\therefore \frac{5.4}{9} = \frac{\boxed{\mathbf{7.5}}}{DF}$$

$\therefore DF = \boxed{\mathbf{12.5}}$.

(ii)



Activity :

$m(\text{arc NS}) = 125°$, $m(\text{arc EF}) = 37°$

$\angle NMS$ has its vertex in the exterior of the circle and intercepts arc EF and arc NS.

$$\therefore \angle NMS = \frac{1}{2}\,[m(\text{arc NS}) - \boxed{\boldsymbol{m}\textbf{(arc EF)}}]$$

$$\therefore \angle NMS = \frac{1}{2} \times [\boxed{\mathbf{125}}° - 37°]$$

$$\therefore \angle NMS = \frac{1}{2} \times \boxed{\mathbf{88°}}$$

$\therefore \angle NMS = \boxed{\mathbf{44°}}$.

(iii) Activity :

$$\frac{\tan^3\theta - 1}{\tan\theta - 1} = \frac{(\tan\theta - \boxed{\mathbf{1}})\,(\tan^2\theta + \boxed{\mathbf{tan}\,\theta} + 1)}{(\tan\theta - 1)}$$

$$= \tan^2\theta + \tan\theta + \boxed{\mathbf{1}}$$

$$= (\tan^2\theta + 1) + \tan\theta$$

$= \boxed{\mathbf{sec^2}\,\theta} + \tan\theta$ … (Trigonometric Identity)

Q. 2. (B) (i) Solution : $7^2 = 49$, $24^2 = 576$, $25^2 = 625$

$7^2 + 24^2 = 49 + 576 = 625$

$\therefore\ 7^2 + 24^2 = 25^2$

$\therefore$ by converse of Pythagoras theorem,

Ans. 7 cm, 24 cm, 25 cm form a right angled triangle.

(ii) Solution : $\square$ MRPN is cyclic ... (Given)

$\therefore$ by theorem of cyclic quadrilateral,

$\angle R + \angle N = 180°$

$\therefore\ (5x - 13)° + (4x + 4)° = 180°$

$\therefore\ 5x - 13 + 4x + 4 = 180$

$\therefore\ 9x - 9 = 180$

$\therefore\ 9x = 180 + 9$

$\therefore\ 9x = 189$

$\therefore\ x = \frac{189}{9}$

$\therefore\ x = 21$

$\angle R = 5x - 13$

$\therefore\ \angle R = 5(21) - 13$

$\therefore\ \angle R = 105 - 13 \qquad \therefore\ \angle R = 92°$

$\angle N = 4x + 4$

$\therefore\ \angle N = 4(21) + 4$

$\therefore\ \angle N = 84 + 4 \qquad \therefore\ \angle N = 88°$

Ans. $\angle R =$ **92°** and $\angle N =$ **88°**.

(iii) Solution : P(0, 6) and Q(12, 20)

Let M(x, y) be the midpoint of seg PQ.

Let P(x_1, y_1) and Q(x_2, y_2)

Here $x_1 = 0$, $y_1 = 6$, $x_2 = 12$, $y_2 = 20$.

By midpoint formula,

$x = \frac{x_1 + x_2}{2}$

$\therefore\ x = \frac{0 + 12}{2}$

$\therefore\ x = \frac{12}{2}$

$\therefore\ x = 6$

$y = \frac{y_1 + y_2}{2}$

$\therefore\ y = \frac{6 + 20}{2}$

$\therefore\ y = \frac{26}{2}$

$\therefore\ y = 13$

Ans. The coordinates of the midpoint of line segment PQ are **(6, 13).**

(iv) Solution : The diameter of the beach ball = 42 cm

$\therefore$ its radius $(r) = \frac{42}{2} = 21$ cm

Surface area of the beach ball $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 21 \times 21$$

$$= 5544 \text{ cm}^2$$

Volume of the beach ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 38808 \text{ cm}^3$$

Ans. Surface area of the beach ball is **5544 cm^2** and volume of the beach ball is **38808 cm^3.**

(v) Solution : $3x = \operatorname{cosec}\theta \qquad \therefore x = \frac{\operatorname{cosec}\theta}{3}$ ... (1)

and $\frac{3}{x} = \cot\theta \qquad \therefore \frac{1}{x} = \frac{\cot\theta}{3}$ ... (2)

$\therefore 3\left(x^2 - \frac{1}{x^2}\right) = 3\left[\frac{\operatorname{cosec}^2\theta}{9} - \frac{\cot^2\theta}{9}\right]$ ... [From (1) and (2)]

$= \frac{3}{9}[\operatorname{cosec}^2\theta - \cot^2\theta]$

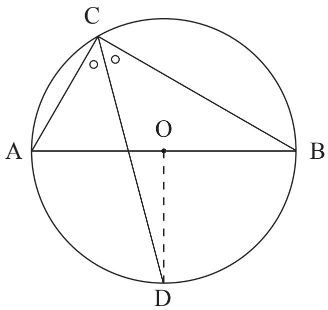
$= \frac{1}{3}(1)$ ... ($\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta$)

$\therefore \operatorname{cosec}^2\theta - \cot\theta = 1$

$= \frac{1}{3}$

Ans. $3\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{3}$.

Q. 3. (A) (i)



Proof :

Draw seg OD

$\angle$ACB = **90°** ... (Angle inscribed in a semicircle)

$\angle DCB =$ **45°** ... (CD is the bisector of $\angle ACB$)

$m(\text{arc } DB) =$ **90°** ... (Inscribed angle theorem)

$\angle DOB =$ **90°** ... (Definition of measure of an arc) ... (1)

seg OA $\cong$ seg OB ... (**Radii of the same circle**) ... (2)

$\therefore$ line OD is **perpendicular bisector** of seg AB the ... [From (1) and (2)]

$\therefore$ seg AD $\cong$ seg BD.

(ii) Activity :

$$\text{Slope of line AB} = \frac{-7-(-2)}{-3-(-4)} = \boxed{\mathbf{-5}}$$

$$\text{Slope of line BC} = \frac{-2-(-7)}{3-(-3)} = \boxed{\mathbf{\frac{5}{6}}}$$

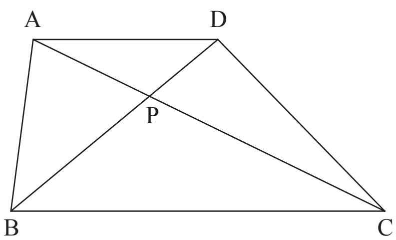
$$\text{Slope of line CD} = \frac{3-(-2)}{2-3} = \boxed{\mathbf{-5}}$$

$$\text{Slope of line AD} = \frac{3-(-2)}{2-(-4)} = \boxed{\mathbf{\frac{5}{6}}}$$

In $\square$ABCD, slopes of opposite sides are **equal**.

$\therefore$ $\square$ABCD is a **parallelogram**.

Q. 3. (B) (i)



Proof : Seg AD $\parallel$ seg BC and line DB is transversal,

$\therefore \angle ADP \cong \angle CBP$ ... (Alternate angles) ... (1)

In $\triangle ADP$ and $\triangle CBP$,

$\angle ADP \cong \angle CBP$ ... [From (1)]

$\angle APD \cong \angle CPB$ ... (Vertically opposite angles)

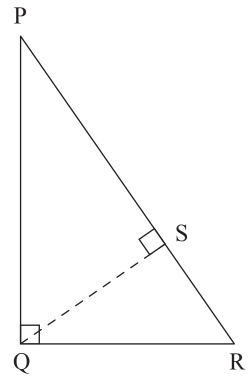
$\therefore \triangle ADP \sim \triangle CBP$ ... (AA test of similarity)

$\therefore \dfrac{AP}{CP} = \dfrac{PD}{BP}$ ... (Corresponding sides of similar triangles are in proportion)

$\therefore \frac{AP}{PD} = \frac{CP}{BP}$ ... (By alternendo)

i.e. $\frac{AP}{PD} = \frac{PC}{BP}$.

(ii)



Given : In $\triangle PQR$, $\angle PQR = 90°$.

To prove : $PR^2 = PQ^2 + QR^2$.

Construction : Draw seg QS $\perp$ side PR such that P–S–R.

Proof : In $\triangle PQR$,

$\angle PQR = 90°$ ... (Given)

Seg QS $\perp$ hypotenuse PR ... (Construction)

$\therefore \triangle PSQ \sim \triangle QSR \sim \triangle PQR$ ... (Similarity of right angled triangles) ... (1)

$\triangle PSQ \sim \triangle PQR$ ... [From (1)]

$\therefore \frac{PS}{PQ} = \frac{PQ}{PR}$ ... (Corresponding sides of similar triangles are in proportion)

$\therefore PQ^2 = PS \times PR$ ... (2)

$\triangle QSR \sim \triangle PQR$ ... [From (1)]

$\therefore \frac{SR}{QR} = \frac{QR}{PR}$ ... (Corresponding sides of similar triangles are in proportion)

$\therefore QR^2 = SR \times PR$ ... (3)

Adding (2) and (3), we get

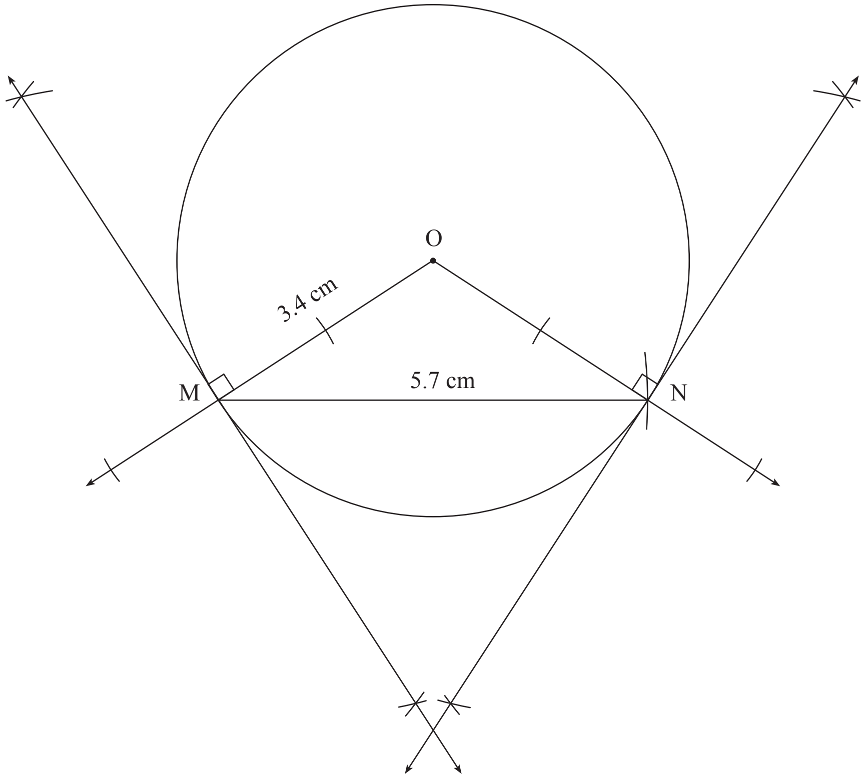
$PQ^2 + QR^2 = PS \times PR + SR \times PR$

$\therefore PQ^2 + QR^2 = PR(PS + SR)$

$\therefore PQ^2 + QR^2 = PR \times PR$ ... (P–S–R)

$\therefore PQ^2 + QR^2 = PR^2$ *OR* $PR^2 = PQ^2 + QR^2$.

(iii)



(iv) Solution : The radius (r) of a metallic sphere = 9 cm.

The diameter of a wire = 4 mm.

$\therefore$ its radius $(r_1) = 2 \text{ mm} = \frac{2}{10}$ cm ... (1 cm = 10 mm)

Let the length be h.

Sphere is melted to make a wire.

$\therefore$ volume of wire = volume of sphere

$\therefore \pi r_1^2 h = \frac{4}{3} \pi r^3$

$\therefore \frac{2}{10} \times \frac{2}{10} \times h = \frac{4}{3} \times 9 \times 9 \times 9$

$\therefore h = \frac{4 \times 3 \times 9 \times 9 \times 10 \times 10}{2 \times 2}$

$\therefore h = 24300$ cm or 243 m ... ($\because$ 1 m = 100 cm)

Ans. Length of the wire is **243 m.**

Q. 4. **(i) Solution :** Diameter of a marble = 1.4 cm.

$\therefore$ its radius $(r) = \frac{1.4}{2} = 0.7 \text{ cm} = \frac{7}{10}$ cm

Diameter of a beaker = 7 cm

its radius $(r_1) = \frac{7}{2}$ cm

Let the number of marbles dropped be x.

When x marbles are dropped, the water rises by 5.6 cm

$h_1 = 5.6 \text{ cm} = \frac{56}{10} \text{ cm}$

Now, Volume of water risen $= x \times$ Volume of each marble

$\pi r_1^2 h_1 = x \times \frac{4}{3} \pi r^3$

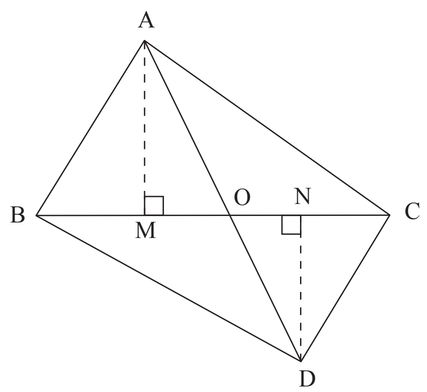
$\frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} = x \times \frac{4}{3} \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$

$\therefore\ x = \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} \times \frac{10}{7} \times \frac{10}{7} \times \frac{10}{7} \times \frac{3}{4}$

$\therefore\ x = 150$

Ans. Number of marbles dropped are **150**.

(ii)



To prove : $\frac{A(\triangle ABC)}{A(\triangle DBC)} = \frac{AO}{DO}$

Proof :

Draw $AM \perp BC$ and $DN \perp BC$.

$\triangle ABC$ and $\triangle DBC$ have same base BC.

$\therefore\ \frac{A\,(\triangle ABC)}{A\,(\triangle DBC)} = \frac{AM}{DN}$ ... (1)

... [Areas of triangles with equal bases are proportional to their corresponding heights]

In $\triangle AMO$ and $\triangle DNO$

$\angle AMO = \angle DNO$ ... (Each measures 90°)

$\angle AOM = \angle DON$ ... (Vertically opposite angles)

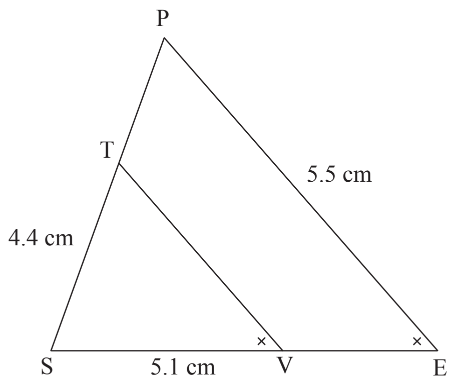
$\triangle AMO \sim \triangle DNO$ ... (AA test of similarity)

$\therefore\ \frac{AM}{DN} = \frac{AO}{DO}$ ... (2) (Corresponding sides of similar triangles)

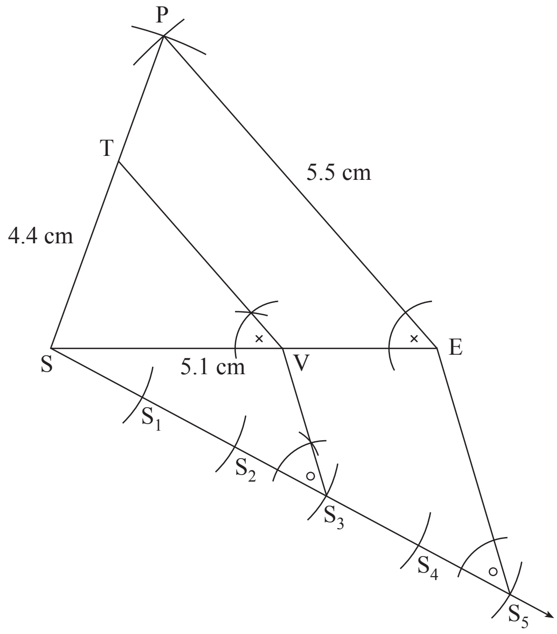
$\therefore$ from (1) and (2), we get

$$\frac{A(\triangle ABC)}{A(\triangle DBC)} = \frac{AO}{DO}.$$

(iii) Ans.



Rough figure



Q. 5. **(i) Solution :**

$$3 \tan\theta = \sec\theta$$

$$\therefore\ 3\ \frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta}$$

$$\therefore\ 3\ \sin\theta = \frac{1}{\cancel{\cos\theta}} \times \cancel{\cos\theta}$$

$$\therefore\ 3\ \sin\theta = 1$$

$$\therefore\ \sin\theta = \frac{1}{3}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\therefore\ \operatorname{cosec}\theta = \frac{3}{1}$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta \qquad \text{... (Trigonometric identity)}$$

$$\therefore\ 3^2 = 1 + \cot^2\theta$$

$$\therefore\ 9 = 1 + \cot^2\theta$$

$$\therefore\ \cot^2\theta = 9 - 1$$

$$\therefore\ \cot^2\theta = 8$$

$$\therefore\ \cot\theta = \sqrt{8}$$

$\therefore\ \cot\theta = 2\sqrt{2}$

Ans. Value of $\cot\theta$ is $\mathbf{2\sqrt{2}}$.

(ii) Proof :

AC = BD ... (Given)

$\therefore$ arc ABC $\cong$ arc DAB ... (Equal chords have their corresponding arcs equal)

m(arc ABC) = m(arc DAB) ... (1)

m(arc ABC) = m(arc BC) + m(arc AB) ... (2)

m(arc DAB) = m(arc AD) + m(arc AB) ... (3) (Arc addition property)

$\therefore$ from (1), (2) and (3)

m(arc BC) + m(arc AB) = m(arc AD) + m(arc AB)

Eliminating m(arc AB) on both the sides, we get

m(arc BC) = m(arc AD)

$\therefore$ BC = AD ... (Congruent arcs have their corresponding chords equal)

i.e. AD = BC