MATHEMATICS (PART–II)

BOARD'S QUESTION PAPER (MARCH 2021)

Time : 2 Hours] **[Total Marks : 40**

[**NOTE :** This examination was not conducted due to Covid-19.]

MATHEMATICS (PART – II)

BOARD'S QUESTION PAPER (SEPTEMBER 2021)

(With Full Solution)

Time : 2 Hours] **[Total Marks : 40**

Note : *(i)* ***All*** *questions are compulsory.*

(ii) Use of calculator is ***not*** *allowed.*

(iii) The numbers to the right of the questions indicate full marks.

(iv) In case of MCQ's [Q. No. ***1(A)****], only the first attempt will be evaluated and will be given credit.*

(v) For every MCQ, the correct alternative (A), (B), (C) or (D) with subquestion number is to be written as an answer.

(vi) Draw proper figures for answers wherever necessary.

(vii) The marks of construction should be clear. Do not erase them.

(viii) Diagram is essential for writing the proof of the theorem.

Q. 1. (A) For each of the following subquestions four alternative answers are given. Choose the correct alternative and write its alphabet : **4**

(i) $\triangle ABC \sim \triangle PQR$. If AB = 4 cm, PQ = 6 cm and QR = 9 cm, then BC = ...

(A) 7 cm (B) 6 cm (C) 8 cm (D) 9 cm

(ii) $\angle PRQ$ is inscribed in the arc PRQ of a circle with centre O. If $\angle PRQ = 75°$, then m(arc PRQ) = ...

(A) 75° (B) 150° (C) 285° (D) 210°

(iii) Seg AB is parallel to Y-axis and coordinates of point A are (1, 3), then coordinates of point B can be ....

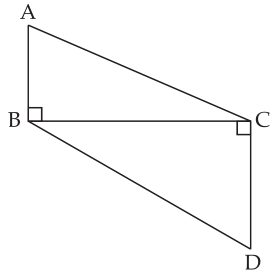
(A) (3, 1) (B) (5, 3) (C) (3, 0) (D) (1, −3)

(iv) Which of the following is not Pythagorean triplet?

(A) (12, 9, 15) (B) (10, 24, 26) (C) (12, 16, 25) (D) (15, 17, 8)

Q. 1. (B) Solve the following subquestions : **4**

(i)



In the above figure, seg AB ⊥ seg BC, seg DC ⊥ seg BC. If AB = 3 and DC = 4, then find $\dfrac{A(\triangle ABC)}{A(\triangle DCB)}$.

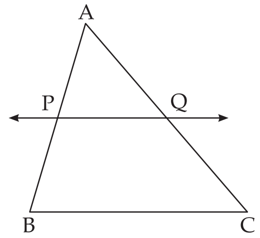
(ii) Find the side of a square whose diagonal is $12\sqrt{2}$ cm.

(iii) If $\tan\theta = \sqrt{3}$, then find the value of θ.

(iv) Radius of the circle with centre C is 6 cm. Line AB is a tangent at point A. What is the measure of $\angle CAB$?

Q. 2. (A) Complete the following activities and rewrite it *(Any two)* : **4**

(i) In $\triangle ABC$, line $PQ \parallel$ side BC. If $AP = 10$, $PB = 12$, $AQ = 15$, then complete the following activity to find the value of QC.



Activity : In $\triangle ABC$, line $PQ \parallel$ side BC ... (Given)

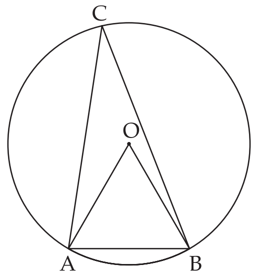
$$\frac{AP}{PB} = \frac{AQ}{QC} \quad \ldots \boxed{}$$

$$\therefore \frac{10}{12} = \frac{\boxed{}}{QC}$$

$$\therefore QC = \frac{\boxed{} \times 12}{10}$$

$$\therefore QC = \boxed{}$$

(ii) In the circle with centre O, length of chord AB is equal to the radius of the circle. Complete the following activity to find measures of $\angle AOB$ and $\angle ACB$.



Activity :

$\angle AOB = \boxed{}^\circ$ ... ($\because \triangle AOB$ is an equilateral triangle)

$\angle ACB = \frac{1}{2} m(\text{arc AB})$ ... $\boxed{}$

$\therefore \angle ACB = \frac{1}{2} \times \boxed{}^\circ$

$\therefore \angle ACB = \boxed{}^\circ$

(iii) To find the distance between the points P(6, −6) and Q(3, −7), complete the following activity.

Activity :

Let $P(6, -6) \equiv (x_1, y_1)$, $Q(3, -7) \equiv (x_2, y_2)$

By distance formula,

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-6)^2 + (-7 - \boxed{})^2}$$

$$= \sqrt{(\boxed{})^2 + (-1)^2}$$

$$= \sqrt{\boxed{} + 1}$$

$$d(P, Q) = \sqrt{\boxed{}}$$

Q. 2. (B) Solve the following subquestions *(Any four)* : **8**

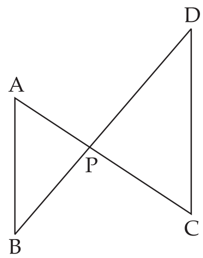
(i) In $\triangle DEF$, $\angle E = 90°$. If DE = 33 cm, DF = 65 cm, then find EF.

(ii) Measures of two arcs formed by a chord of a circle are $2x°$ and $7x°$. Find the measure of minor arc.

(iii) If A(−7, 6), B(2, −2) and C(8, 5) are the coordinates of vertices of a triangle, then find the coordinates of centroid.

(iv) If $\sin\theta = \frac{7}{25}$, then find the value of $\cos\theta$.

(v)



In the above figure, seg AC and seg BD intersect each other in point P and $\frac{AP}{CP} = \frac{BP}{DP}$, then prove that $\triangle ABP \sim \triangle CDP$.

Q. 3. (A) Complete the following activities and rewrite it *(Any one)* : **3**

(i) If $\triangle ABC \sim \triangle PQR$, $A(\triangle ABC) = 81\ cm^2$, $A(\triangle PQR) = 121\ cm^2$, BC = 6.3 cm, then complete the following activity to find QR.

Activity :

$\triangle ABC \sim \triangle PQR$ ... (Given)

$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{\boxed{}}{QR^2}$ ... $\boxed{}$

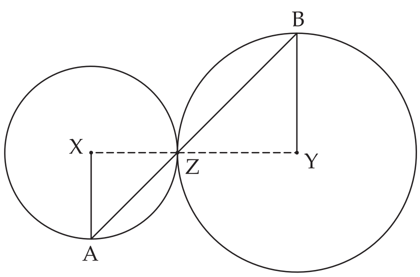
$\therefore \frac{\boxed{}}{121} = \frac{(6.3)^2}{QR^2}$

$\therefore \frac{\boxed{}}{11} = \frac{6.3}{QR}$ ... (Taking square roots of both sides) ... (I)

$\therefore QR = \frac{6.3 \times 11}{\boxed{}}$

$\therefore QR = \boxed{}$ cm

(ii)



In the above figure circles with centres X and Y touch each other at point Z. A secant passing through Z intersects the circles at points A and B respectively.

Complete the following activity to prove radius $XA \parallel$ radius YB.

Activity : Draw segments XZ and seg ZY.

$\therefore$ by theorem of touching circles points X, Z, Y are ☐

$\therefore \angle XZA \cong$ ☐ ... (1) ... (Vertically opposite angles)

Now seg XA $\cong$ seg XZ ... ☐

$\therefore \angle XAZ \cong$ ☐ ... (2) ... (Isosceles triangle theorem)

Similarly, seg YB $\cong$ seg YZ

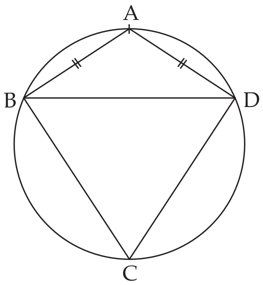
$\therefore \angle BZY \cong \angle YBZ$ ... (3)

$\therefore \angle XAZ =$ ☐ ... [From (1), (2) and (3)]

$\therefore$ radius $XA \parallel$ radius YB ... ☐

Q. 3. (B) Solve the following subquestions (*Any two)* : **6**

(i) Prove that, "In a right-angled triangle, the perpendicular segment to the hypotenuse from the opposite vertex, is the geometric mean of the segments into which the hypotenuse is divided."

(ii) 

□ABCD is cyclic, AB = AD, $\angle BCD = 70°$, then find :

(a) m(arc BCD)

(b) m(arc BAD)

(c) $\angle ABD$.

(iii) Draw a circle with centre P and radius 3.5 cm. Draw an arc AB of 120° measure. Draw tangents to the circle at point A and point B.

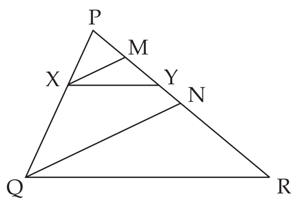
(iv) Prove that

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \text{cosec } A - \cot A.$$

Q. 4. Solve the following subquestions *(Any two)* : **8**

(i) If two consecutive angles of a cyclic quadrilateral are congruent, then prove that one pair of opposite sides is parallel and other pair is congruent.

(ii) $\triangle LMN \sim \triangle LQP$. In $\triangle LMN$, LM = 3.6 cm, $\angle L = 50°$, LN = 4.2 cm and $\frac{LM}{LQ} = \frac{4}{7}$, then construct $\triangle LQP$ and $\triangle LMN$.

(iii) 

In $\triangle PQR$, seg $XY \parallel$ side QR, point M and point N are midpoints of seg PY and seg PR respectively, then prove that :

(a) $\triangle PXM \sim \triangle PQN$

(b) seg $XM \parallel$ seg QN.

Q. 5. Solve the following subquestions *(Any one)* : **3**

(i) Draw the $\angle ABC$ of measure $65°$. Draw ray BM which is a bisector of $\angle B$. Take point P on ray BM such that $BP = 4$ cm. Draw perpendicular on arm BC through the point P. Draw a circle with centre P and length of perpendicular as a radius. Write the measure of radius. Observe the figure and write the relation between circle and arms of the angle.

(ii) If point P divides the seg AB joining the points A(2, 1) and B(-3, 6) in the ratio 2 : 3, then determine whether the point P lies on the line $x - 5y + 15 = 0$ or not.

SOLUTION : BOARD'S QUESTION PAPER (SEPTEMBER 2021)

Q. 1. (A) (i) (B)

(ii) (D)

(iii) (D)

(iv) (C)

Hints : Only for guidance. Students are not expected to write this.

(i) (B) Corresponding sides of similar triangles are proportional, $\frac{AB}{PQ} = \frac{BC}{QR}$.

(ii) (D) Same as (ii) of November 2020.

(iii) (D) Seg AB $\parallel$ Y-axis. $\therefore$ x-coordinate for points A and B is the same.

(iv) (C) (A), (B) and (D) are Pythagorean triplets.

(C) is not a Pythagorean triplet.

Q. 1. (B) **(i)** $\triangle ABC$ and $\triangle DCB$ have equal bases.

$\therefore \frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{AB}{DC}$ ... (Triangles having equal bases)

$\therefore \frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{3}{4}$

(ii) Diagonal of a square $= \sqrt{2} \times$ side

$\therefore 12\sqrt{2} = \sqrt{2} \times$ side

$\therefore$ side $= \frac{12\sqrt{2}}{\sqrt{2}}$

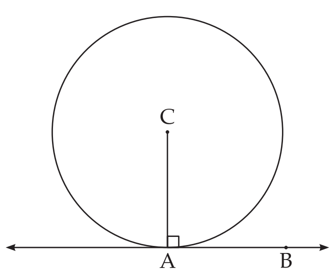
$\therefore$ side $= 12$ cm

(iii) $\tan\theta = \sqrt{3}$

$\therefore \tan 60 = \sqrt{3}$

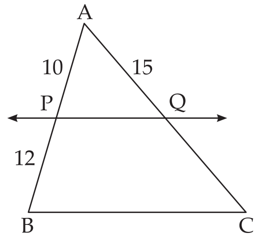
$\therefore \theta = 60°$

(iv)



$\angle CAB = 90°$ ... (Tangent theorem)

Q. 2. (A) (i)



Activity :

In $\triangle ABC$,

seg PQ $\parallel$ side BC ... (Given)

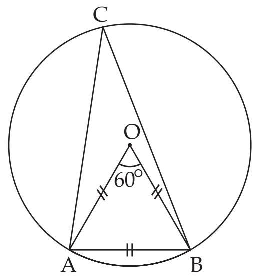
$\frac{AP}{PB} = \frac{AQ}{QC}$ ... **(Basic Proportionality theorem)**

$\therefore \frac{10}{12} = \frac{\boxed{15}}{QC}$

$\therefore QC = \frac{\boxed{15} \times 12}{10}$

$\therefore QC = \boxed{18}$

(ii)



Activity :

$\angle AOB = \boxed{60°}$ ... ($\because$ $\triangle AOB$ is an equilateral triangle)

$\angle ACB = \frac{1}{2} m(\text{arc AB})$ ... **(Inscribed angle theorem)**

$\therefore \angle ACB = \frac{1}{2} \times \boxed{60°}$

$\therefore \angle ACB = \boxed{30°}$

(iii) Activity :

Let $P(6, -6) \equiv (x_1, y_1)$

$Q(3, -7) \equiv (x_2, y_2)$

By distance formula,

$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

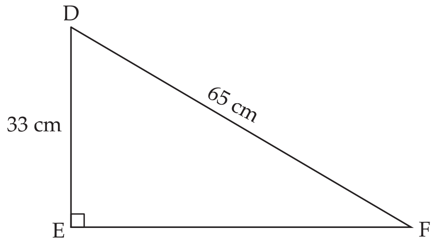
$= \sqrt{(3-6)^2 + (-7 - \boxed{-6})^2}$

$= \sqrt{(\boxed{-3})^2 + (-1)^2}$

$= \sqrt{\boxed{9} + 1}$

$= \sqrt{\boxed{10}}$

Q. 2. (B) (i)



Solution :

In $\triangle DEF$,

$\angle DEF = 90°$

$\therefore$ by Pythagoras theorem,

$DF^2 = DE^2 + EF^2$

$\therefore 65^2 = 33^2 + EF^2$

$\therefore EF^2 = 65^2 - 33^2$

$\therefore EF^2 = 4225 - 1089$

$\therefore EF^2 = 3136$

$\therefore EF = \sqrt{3136}$

$\therefore EF = 56$ cm

Ans. EF = **56 cm.**

(ii) Solution :

Measure of minor arc $= 2x°$

Measure of major arc $= 7x°$

$\therefore 2x° + 7x° = 360°$ ... (Measure of a circle)

$\therefore 9x = 360°$

$\therefore x = \frac{360}{9}$

$\therefore x = 40°$

Measure of minor arc $= 2x$

$= 2 \times 40°$

$= 80°$

Ans. Measure of minor arc is **80°.**

(iii) Solution :

$A(-7, 6) = (x_1, y_1)$

$B(2, -2) = (x_2, y_2)$

$C(8, 5) = (x_3, y_3)$

By centroid formula,

$x = \frac{x_1 + x_2 + x_3}{3}, \quad y = \frac{y_1 + y_2 + y_3}{3}$

$x = \dfrac{-7 + 2 + 8}{3}$, $y = \dfrac{6 - 2 + 5}{3}$

$x = \dfrac{3}{3}$, $y = \dfrac{9}{3}$

$x = 1$, $y = 3$

Ans. Coordinates of centroid are **(1, 3).**

(iv) Solution :

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \left(\dfrac{7}{25}\right)^2 + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \left(\dfrac{7}{25}\right)^2$

$\therefore \cos^2\theta = 1 - \dfrac{49}{625}$

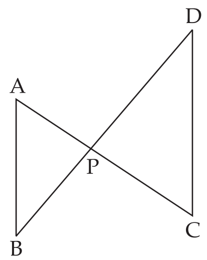
$\therefore \cos^2\theta = \dfrac{625 - 49}{625}$

$\therefore \cos^2\theta = \dfrac{576}{625}$

$\therefore \cos\theta = \dfrac{24}{25}$ ... (Taking square roots of both the sides)

Ans. $\cos\theta = \mathbf{\dfrac{24}{25}}$**.**

(v) Proof :



In $\triangle ABP$ and $\triangle CDP$,

$\dfrac{AP}{CP} = \dfrac{BP}{DP}$ ... (Given)

$\angle APB = \angle CPD$ ... (Vertically opposite angles)

$\therefore \triangle ABP \sim \triangle CDP$ ... (SAS test of similarity)

Q. 3. (A) (i) Activity :

$\triangle ABC \sim \triangle PQR$ ... (Given)

$\dfrac{A(\triangle ABC)}{A(\triangle PQR)} = \dfrac{\boxed{\mathbf{BC^2}}}{QR^2}$ ... (**Theorem of areas of similar triangles**)

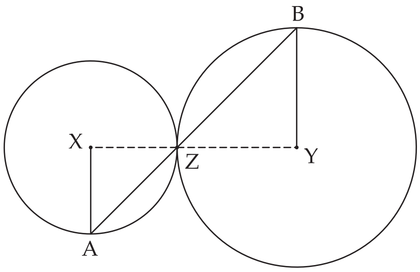
$\therefore \dfrac{\boxed{\mathbf{81}}}{121} = \dfrac{(6.3)^2}{QR^2}$

$\therefore \dfrac{\boxed{\mathbf{9}}}{11} = \dfrac{6.3}{QR}$ ... (Taking square root of both sides)

$\therefore QR = \dfrac{11 \times 6.3}{\boxed{\mathbf{9}}}$

$\therefore QR = \boxed{\mathbf{7.7}}$ cm

(ii)



Activity :

Draw segments XZ and ZY.

$\therefore$ by theorem of touching circles points X, Y, Z are $\boxed{\textbf{collinear}}$

$\therefore \angle XZA \cong \boxed{\angle \mathbf{YZB}}$ ... Vertically opposite angles ... (1)

Now seg XA $\cong$ seg XZ ... ($\boxed{\textbf{Radii of same circle}}$)

$\therefore \angle XAZ \cong \boxed{\angle \mathbf{XZA}}$ ... (Isosceles triangle theorem) ... (2)

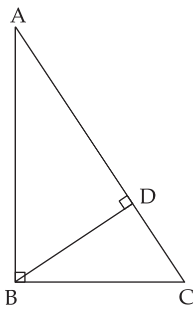
Similarly, seg YB $\cong$ seg YZ

$\therefore \angle BZY \cong \angle YBZ$ ... (3)

$\therefore \angle XAZ = \boxed{\angle \mathbf{YBZ}}$ ... [From (1), (2) and (3)]

$\therefore$ radius XA $\parallel$ radius YB ... ($\boxed{\textbf{Alternate angles test}}$)

Q. 3. (B) **(i)** **Statement :** In a right angled triangle, the perpendicular segment to the hypotenuse from the opposite vertex, is the geometric mean of the segments into which the hypotenuse is divided.



Given : In $\triangle ABC$, $\angle ABC = 90°$.

seg BD $\perp$ hypotenuse AC such that A-D-C.

To prove : $BD^2 = AD \times DC$.

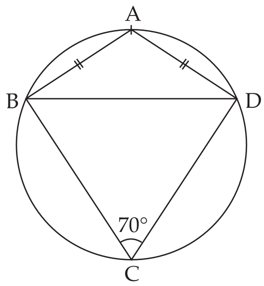
Proof : In $\triangle ABC$, $\angle ABC = 90°$ ... (Given) seg BD $\perp$ hypotenuse AC

$\therefore$ $\triangle ADB \sim \triangle BDC$ ... (Similarity of right angled triangles)

$\therefore$ $\frac{AD}{BD} = \frac{BD}{DC}$ ... (Corresponding sides of similar triangles are in proportion)

$\therefore$ $BD^2 = AD \times DC$.

(ii)



Solution :

$\square$ABCD is a cyclic quadrilateral.

$\angle BAD + \angle BCD = 180°$

... (Opposite angles of cyclic quadrilateral are supplementary)

$\therefore$ $\angle BAD + 70° = 180°$

$\angle BAD = 180° - 70°$

$\angle BAD = 110°$

$\angle BAD = \frac{1}{2} m(\text{arc BCD})$ ... (Inscribed angle theorem)

$\therefore$ $110° = \frac{1}{2} m(\text{arc BCD})$

$\therefore$ $m(\text{arc BCD}) = 110° \times 2$

$\therefore$ $m(\text{arc BCD}) = 220°$

$\angle BCD = \frac{1}{2} m(\text{arc BAD})$ ... (Inscribed angle theorem)

$70 = \frac{1}{2} m(\text{arc BAD})$

$\therefore$ $m(\text{arc BAD}) = 2 \times 70°$

$m(\text{arc BAD}) = 140°$

In $\triangle ABD$,

seg AB $\cong$ seg AD ... (Given)

$\angle ABD = \angle ADB$ ... (Isosceles triangle theorem)

$\angle BAD + \angle ABD + \angle ADB = 180°$

... (Sum of all angles of triangle is 180°)

$110° + \angle ABD + \angle ABD = 180°$

$2\angle ABD = 180° - 110°$

$2\angle ABD = 70°$

$\angle ABD = \frac{70}{2}$

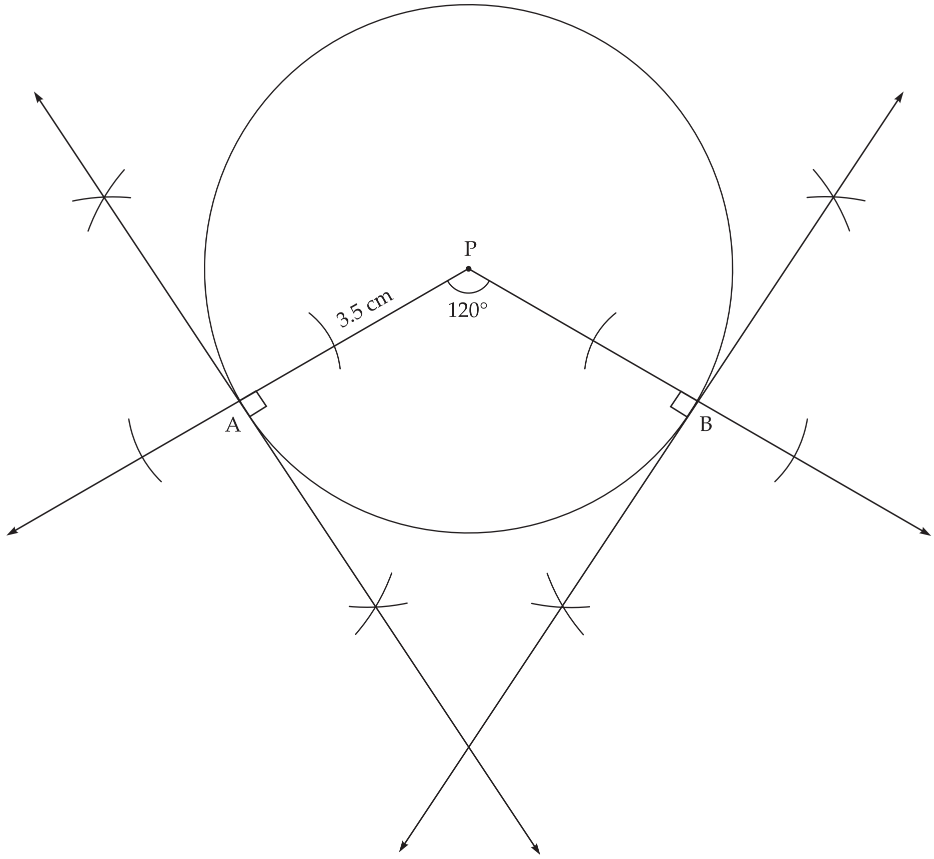
$\angle ABD = 35°$

Ans. (a) m (arc BCD) $=$ **220°**

(b) m (arc BAD) $=$ **140°**

(c) $\angle ABD =$ **35°**.

(iii) Ans.



Steps of construction :

(1) Draw a circle of radius 3.5 cm.

(2) Draw arc AB of 120° measure.

(3) Draw tangent through point A.

(4) Draw tangent through point B.

(iv) Proof :

$$\text{LHS} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$= \sqrt{\frac{(1 - \cos A)}{(1 + \cos A)} \times \frac{(1 - \cos A)}{(1 - \cos A)}}$$

$$= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}}$$

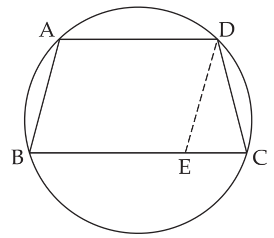
$$= \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} \qquad \left[\begin{array}{l} \because \sin^2 A + \cos^2 A = 1 \\ \therefore 1 - \cos^2 A = \sin^2 A \end{array}\right]$$

$$= \frac{1 - \cos A}{\sin A}$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \operatorname{cosec} A - \cot A$$

LHS = RHS

Q. 4. (i)



Given : $\square$ABCD is cyclic and $\angle B = \angle C$

To prove : (i) seg AD $\parallel$ seg BC

(ii) seg AB $\cong$ seg DC

Construction : Draw seg DE $\parallel$ seg AB

Proof :

$\square$ABCD is cyclic.

$\therefore$ $\angle A + \angle C = 180°$ ... (1) (Opposite angles of cyclic quadrilateral)

But $\angle B = \angle C$ ... (2) (Given)

$\therefore$ $\angle A + \angle B = 180°$ ... [From (1) and (2)]

$\therefore$ seg AD $\parallel$ seg BC ... (Interior angles test)

Now, seg AB $\parallel$ seg DE ... (Construction)

$\therefore$ $\square$ABED is a parallelogram.

$\therefore$ AB = DE ... (3) (Opposite sides of parallelogram)

Similarly, $\angle B = \angle DEC$ ... (4) (Corresponding angles)

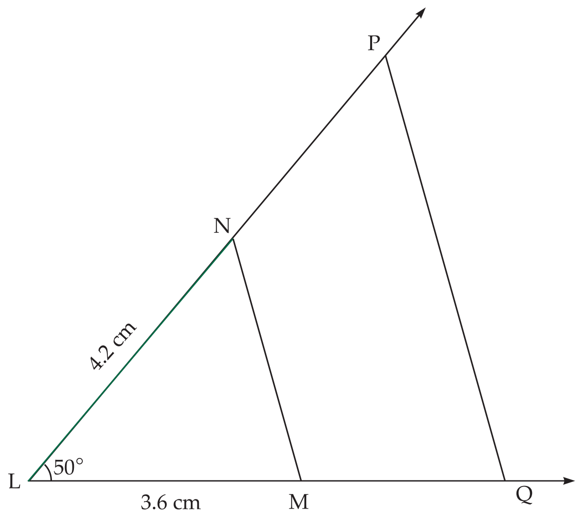
$\therefore$ $\angle C = \angle DEC$ ... [From (2) & (4)]

$\therefore$ DE = DC ... (5) (sides opposite to congruent angles)

$\therefore$ AB = DC ... [From (3) & (5)]

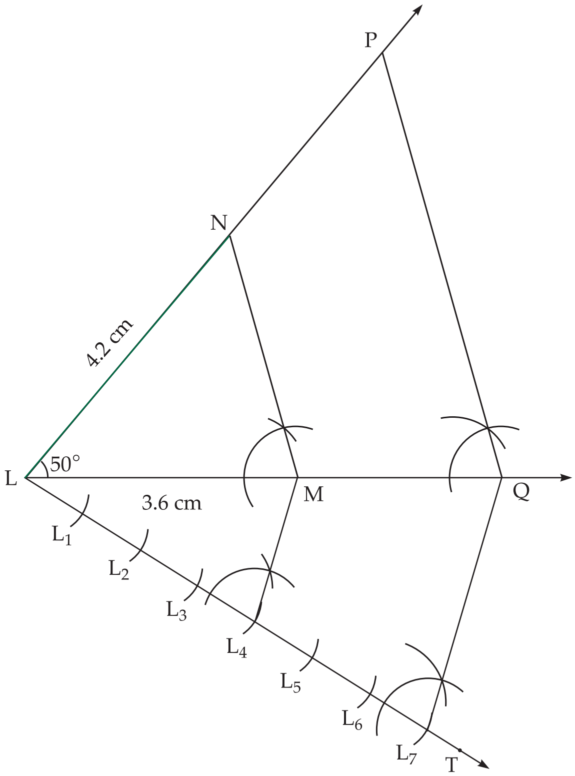
(ii)

Analytical figure :



Ans.

$\frac{LM}{LQ} = \frac{4}{7}$



Steps of construction :

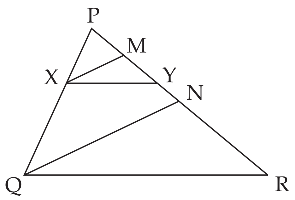
(1) Draw $\triangle$LMN

(2) Draw ray LT and to show 7 equal parts on it.

(3) Draw seg $L_7Q \parallel L_4M$

(4) Draw seg QP.

(iii)



Proof :

seg XY $\parallel$ seg QR and PQ is the transversal

$\therefore \angle PXY \cong \angle PQR$ ... (Corresponding angles) ... (1)

In $\triangle PXY$ and $\triangle PQR$,

$\angle PXY \cong \angle PQR$ ... [From (1)]

$\angle XPY \cong \angle QPR$ ... (Common angle)

$\therefore \triangle PXY \sim \triangle PQR$ ... (AA test of similarity)

$\therefore \frac{PX}{PQ} = \frac{PY}{PR}$ ... (2) ... (Corresponding sides of similar triangles)

$PY = 2PM$ ... (3) ... (M is the midpoint of PY)

$PR = 2PN$ ... (4) ... (N is the midpoint of PR)

$\therefore \frac{PX}{PQ} = \frac{2PM}{2PN}$ ... [From (2), (3) and (4)]

$\therefore \frac{PX}{PQ} = \frac{PM}{PN}$ ... (5)

In $\triangle PXM$ and $\triangle PQN$,

$\frac{PX}{PQ} = \frac{PM}{PN}$ ... [From (5)]

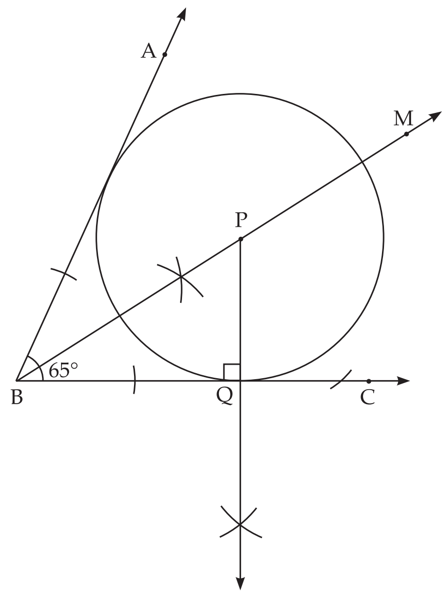
$\angle XPM \cong \angle QPN$ ... (Common angle)

$\therefore \triangle PXM \sim \triangle PQN$ ... (SAS test of similarity)

$\therefore \angle PXM \cong \angle PQN$ ... (Corresponding angles of similar triangles)

$\therefore$ seg QM $\parallel$ seg QN ... (Corresponding angle test)

Q. 5. (i) Ans.



Steps of construction :

(1) Draw $\angle ABC = 65°$

(2) Draw bisector of $\angle ABC$ and locating point P such that BP $= 4$ cm

(3) Draw $PQ \perp BC$.

(4) Draw circle with centre P and radius PQ.

(5) Measure of radius $= 2.1$ cm.

(6) The sides of the angle are tangents to the circle.

(ii) Solution :

Let $A \equiv (2, 1) \equiv (x_1, y_1)$

$B \equiv (-3, 6) \equiv (x_2, y_2)$

P(x, y) divides AB in the ratio $m : n$.

$m : n = 2 : 3$

By section formula,

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\therefore\ x = \frac{2 \times (-3) + 3 \times 2}{2 + 3}$$

$$\therefore\ x = \frac{-6 + 6}{5}$$

$$\therefore\ x = \frac{0}{5}$$

$$\therefore\ x = 0$$

$$y = \frac{my_2 + ny_1}{m + n}$$

$$\therefore\ y = \frac{2 \times 6 + 3 \times 1}{2 + 3}$$

$$\therefore\ y = \frac{12 + 3}{5}$$

$$\therefore\ y = \frac{15}{5}$$

$$\therefore\ y = 3$$

$$\therefore\ (x, y) = (0, 3)$$

Now to determine whether point P lies on line $x - 5y + 15 = 0$

We substitute $x = 0$ and $y = 3$ in LHS.

$$\begin{aligned} \text{LHS} &= x - 5y + 15 \\ &= 0 - 5(3) + 15 \\ &= 0 - 15 + 15 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ LHS = RHS

The coordinate of point P satisfy the equation.

Ans. P **lies** on the line $x - 5y + 15 = 0$.

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