MATHEMATICS (PART – II)

BOARD'S QUESTION PAPER (MARCH 2020)

(With Full Solution)

Time : 2 Hours] **[Total Marks : 40**

Note : *(i)* ***All*** *questions are compulsory.*

(ii) Use of calculator is ***not*** *allowed.*

(iii) The numbers to the right of the questions indicate full marks.

(iv) In case of MCQ's [Q. No. ***1(A)****], only the first attempt will be evaluated and will be given credit.*

(v) For every MCQ, the correct alternative (A), (B), (C) or (D) in front of subquestion number is to be written as an answer.

(vi) Draw proper figures for answers wherever necessary.

(vii) The marks of construction should be clear and distinct. Do not erase them.

(viii) Diagram is essential for writing the proof of the theorem.

Q. 1. (A) Four alternative answers are given for each of the following subquestions. Choose the correct alternative and write the letter of the alphabet of that answer : **4**

(i) Out of the following which is the Pythagorean triplet?

(A) (1, 5, 10) (B) (3, 4, 5) (C) (2, 2, 2) (D) (5, 5, 2)

(ii) Two circles of radii 5.5 cm and 3.3 cm respectively touch each other externally. What is the distance between their centres?

(A) 4.4 cm (B) 2.2 cm (C) 8.8 cm (D) 8.9 cm

(iii) Distance of point $(-3, 4)$ from the origin is ... .

(A) 7 (B) 1 (C) -5 (D) 5

(iv) Find the volume of a cube of side 3 cm.

(A) 27 cm^3 (B) 9 cm^3 (C) 81 cm^3 (D) 3 cm^3

Q. 1. (B) Solve the following subquestions : **4**

(i) The ratio of corresponding sides of similar triangles is 3 : 5, then find the ratio of their areas.

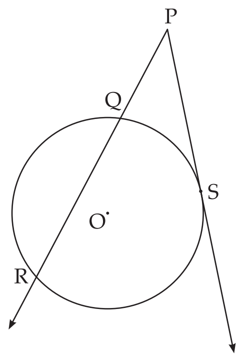
(ii) Find the diagonal of a square whose side is 10 cm.

(iii) $\square$ABCD is cyclic. If $\angle B = 110°$, then find the measure of $\angle D$.

(iv) Find the slope of the line passing through the points A(2, 3) and B(4, 7).

Q. 2. (A) Complete and write *any two* of the following activities : **4**

(i)



In the figure given above, O is the centre of the circle, seg PS is a tangent segment and S is the point of contact. Line PR is a secant.

If PQ = 3.6, QR = 6.4, find PS.

Activity :

$PS^2 = PQ \times \square$ ... (Tangent secant segments theorem)

$= PQ \times \left(PQ + \square\right)$

$= 3.6 \times (3.6 + 6.4)$

$= 3.6 \times \square$

$= 36$

$PS = \square$ ... (By taking square roots)

(ii) If $\sec\theta = \dfrac{25}{7}$, complete the following activity to find the value of $\tan\theta$.

Activity :

$1 + \tan^2\theta = \sec^2\theta$

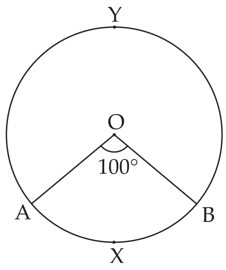
$\therefore 1 + \tan^2\theta = \left(\dfrac{25}{7}\right)^{\square}$

$\therefore \tan^2\theta = \dfrac{625}{49} - \square$

$= \dfrac{625 - 49}{49} = \dfrac{\square}{49}$

$\therefore \tan\theta = \dfrac{\square}{7}$ ... (By taking square roots)

(iii)



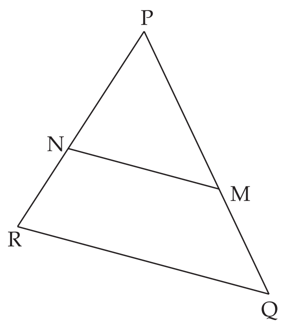
In the figure given above, O is the centre of the circle. Using the given information complete the following table :

Activity :

Type of arc	Name of the arc	Measure of the arc
Minor arc		
Major arc		

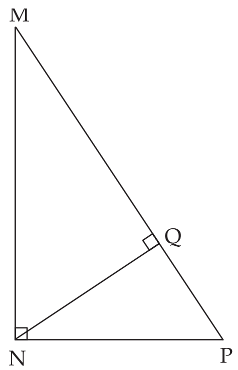
Q. 2. (B) Solve *any four* of the following subquestions : 8

(i)

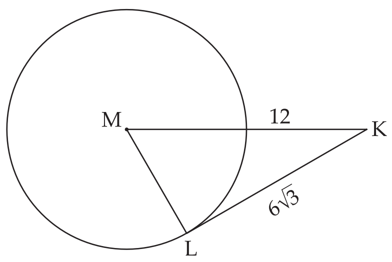


In $\triangle PQR$, $NM \parallel RQ$. If $PM = 15$, $MQ = 10$, $NR = 8$, then find PN.

(ii) In $\triangle MNP$, $\angle MNP = 90°$, seg $NQ \perp$ seg MP. If $MQ = 9$, $QP = 4$, then find NQ.



(iii)



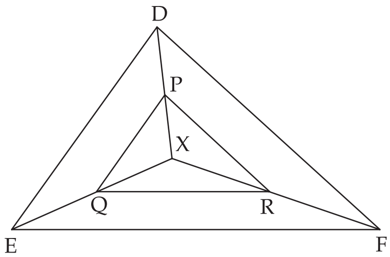
In the given figure, M is the centre of the circle and seg KL is a tangent segment. L is a point of contact. If $MK = 12$, $KL = 6\sqrt{3}$, then find the radius of the circle.

(iv) Find the coordinates of the midpoint of the segment joining the points (22, 20) and (0, 16).

(v) A person is standing at a distance of 80 metres from a Church and looking at its top. The angle of elevation is of 45°. Find the height of the Church.

Q. 3. (A) Complete and write *any one* of the following activities : **3**

(i) In the given figure, X is any point in the interior of the triangle. Point X is joined to the vertices of the triangle. seg PQ ∥ seg DE, seg QR ∥ seg EF. Complete the activity and prove that seg PR ∥ seg DF.



Activity :

In $\triangle XDE$,

PQ ∥ DE ... (Given)

$\therefore \frac{XP}{PD} = \frac{\square}{QE}$ ... (Basic proportionality theorem) ... (I)

In $\triangle XEF$,

QR ∥ EF ... (Given)

$\therefore \frac{XQ}{\square} = \frac{XR}{\square}$ ... ($\square$) ... (II)

$\therefore \frac{XP}{PD} = \frac{\square}{\square}$ ... [From (I) and (II)]

$\therefore$ seg PR ∥ seg DF ... (By converse of basic proportionality theorem)

(ii) If A(6, 1), B(8, 2), C(9, 4) and D(7, 3) are the vertices of □ABCD, show that □ABCD is a parallelogram.

Activity : Slope of line $= \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ slope of line AB $= \frac{2-1}{8-6} = \square$ ... (I)

$\therefore$ slope of line BC $= \frac{4-2}{9-8} = \square$ ... (II)

$\therefore$ slope of line CD $= \frac{3-4}{7-9} = \square$ ... (III)

$\therefore$ slope of line DA $= \frac{3-1}{7-6} = \square$ ... (IV)

$\therefore$ slope of line AB $= \square$ ... [From (I) and (III)]

line AB ∥ line CD

$\therefore$ slope of line BC $= \square$ ... [From (II) and (IV)]

line BC ∥ line DA

Both the pairs of opposite sides of the quadrilateral are parallel.

$\therefore$ □ABCD is a parallelogram.

Q. 3. (B) Solve *any two* of the following subquestions : **6**

(i) In $\triangle PQR$, point S is the midpoint of side QR. If $PQ = 11$, $PR = 17$, $PS = 13$, find QR.

(ii) Prove that, tangent segments drawn from an external point to the circle are congruent.

(iii) Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre.

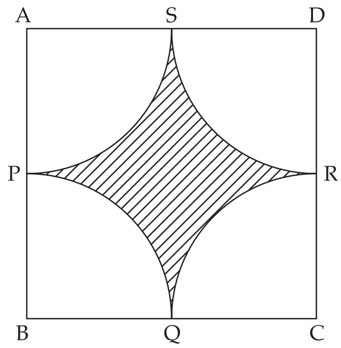
(iv) A metal cuboid of measures 16 cm $\times$ 11 cm $\times$ 10 cm was melted to make coins. How many coins were made, if the thickness and diameter of each coin was 2 mm and 2 cm respectively?

Q. 4. Solve *any two* of the following subquestions : **8**

(i) In $\triangle ABC$, PQ is a line segment intersecting AB at P and AC at Q such that seg PQ $\parallel$ seg BC. If PQ divides $\triangle ABC$ into two parts having equal areas, find $\frac{BP}{AB}$.

(ii) Draw a circle with radius 2.7 cm and draw a chord PQ of length 4.5 cm. Draw tangents at points P and Q without using centre.

(iii) In the given figure, $\square$ABCD is a square of side 50 m. Points P, Q, R, S are midpoints of side AB, side BC, side CD, side AD respectively. Find the area of the shaded region.



Q. 5. Solve *any one* of the following subquestions : **3**

(i) Circles with centres A, B and C touch each other externally. If $AB = 3$ cm, $BC = 3$ cm, $CA = 4$ cm, then find the radius of each circle.

(ii) If $\sin\theta + \sin^2\theta = 1$
show that $\cos^2 + \cos^4\theta = 1$.

SOLUTION : BOARD'S QUESTION PAPER (MARCH 2020)

Q. 1. (A) (i) (B)

(ii) (C)

(iii) (D)

(iv) (A)

Hints : Only for guidance. Students are not expected to write this.

(i) (B) Pythagorean triplet : $a^2 + b^2 = c^2$. 3, 4, 5. $5^2 = 3^2 + 4^2$.

(ii) (C) The distance between the centre of circles touching externally $= r_1 + r_2$.

(iii) (D) Using Pythagorean triplet, 3, 4, 5. Distance is always positive.

(iv) (A) Volume of a cube $=$ (side)^3.

Q. 1. (B) **(i)** Let the corresponding sides of two similar triangles be S_1 and S_2 respectively.

Let their respective areas be A_1 and A_2.

$\frac{S_1}{S_2} = \frac{3}{5}$ ... (Given)

The triangles are similar.

$\frac{A_1}{A_2} = \frac{S_1^2}{S_2^2}$ ... (Theorem of areas of similar triangles)

$\therefore \frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$

$\therefore \frac{A_1}{A_2} = \left(\frac{3}{5}\right)^2$

$\therefore \frac{A_1}{A_2} = \frac{9}{25}$

$\therefore A_1 : A_2 =$ **9 : 25**

(ii) Side of square $= 10$ cm

Diagonal of a square $= \sqrt{2} \times \text{side}$

$= \sqrt{2} \times 10$

$= 10\sqrt{2}$ cm

Diagonal of the square is **$10\sqrt{2}$ cm.**

(iii) $\square$ABCD is cyclic.

$\therefore \angle B + \angle D = 180°$ ... (Opposite angles of a cyclic quadrilateral are supplementary)

$\therefore 110° + \angle D = 180°$

$\therefore \angle D = 180° - 110°$

$\therefore \angle D =$ **70°**

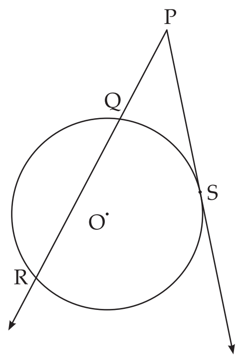
(iv) Slope of line AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{7-3}{4-2}$

$= \dfrac{4}{2} = 2$

Slope of line AB is **2.**

Q. 2. (A) (i)



Activity :

$PS^2 = PQ \times \boxed{\mathbf{PR}}$ ... (Tangent secant segments theorem)

$= PQ \times (PQ + \boxed{\mathbf{QR}})$

$= 3.6 \times (3.6 + 6.4)$

$= 3.6 \times \boxed{\mathbf{10}}$

$= 36$

$\therefore PS = \boxed{\mathbf{6}}$ ... (By taking square roots)

(ii) Activity :

$1 + \tan^2\theta = \sec^2\theta$

$\therefore\ 1 + \tan^2\theta = \left(\dfrac{25}{7}\right)^{\boxed{\mathbf{2}}}$

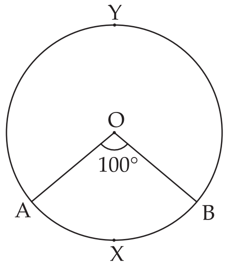
$\therefore\ \tan^2\theta = \dfrac{625}{49} - \boxed{\mathbf{1}}$

$= \dfrac{625 - 49}{49}$

$= \dfrac{\boxed{\mathbf{576}}}{49}$

$\therefore \tan\theta = \dfrac{\boxed{\mathbf{24}}}{7}$ ... (By taking square roots)

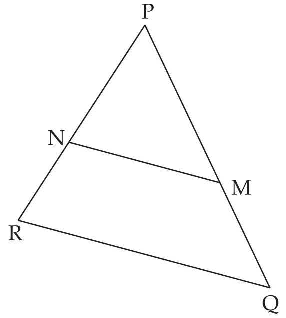
(iii)



Activity :

Type of arc	Name of the arc	Measure of the arc
Minor arc	**arc AXB**	**100°**
Major arc	**arc AYB**	**260°**

Q. 2. (B) (i) Solution :



In $\triangle PRQ$,

seg NM $\parallel$ side RQ ... (Given)

$\therefore$ by Basic Proportionality theorem,

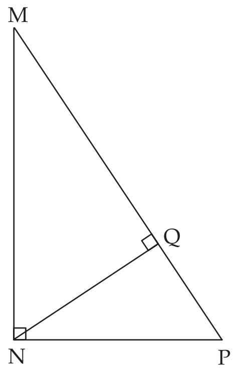
$$\frac{PN}{NR} = \frac{PM}{MQ}$$

$$\therefore \frac{PN}{8} = \frac{15}{10}$$

$$\therefore PN = \frac{15 \times 8}{10}$$

Ans. PN = **12**

(ii) Solution :



In $\triangle MNP$,

$\angle MNP = 90°$, seg NQ $\perp$ seg MP

$\therefore$ by theorem of geometric mean,

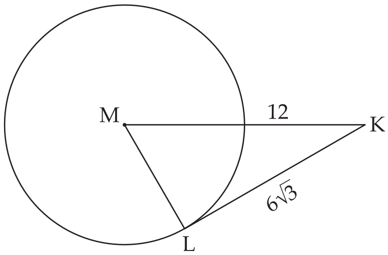
$NQ^2 = MQ \times QP$

$\therefore$ $NQ^2 = 9 \times 4$

$\therefore$ $NQ = 3 \times 2$ ... (Taking square roots)

Ans. $NQ = \mathbf{6}$

(iii) Solution :



In $\triangle MLK$,

$\angle MLK = 90°$ ... (Tangent theorem)

$\therefore$ by Pythagoras theorem,

$MK^2 = ML^2 + LK^2$

$\therefore$ $12^2 = ML^2 + (6\sqrt{3})^2$

$\therefore$ $144 = ML^2 + 36 \times 3$

$\therefore$ $144 = ML^2 + 108$

$\therefore$ $ML^2 = 144 - 108$

$\therefore$ $ML^2 = 36$

$\therefore$ $ML = 6$ ... (Taking square roots)

Ans. Radius of the circle is **6.**

(iv) Solution : Let $A(22, 20) = (x_1, y_1)$

$B(0, 16) = (x_2, y_2)$

Let $M(x, y)$ be the midpoint of AB.

$\therefore$ by midpoint formula,

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

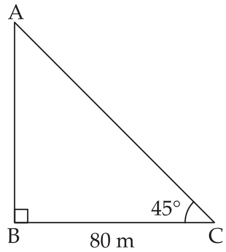
$$\therefore (x, y) = \left(\frac{22 + 0}{2}, \frac{20 + 16}{2}\right)$$

$$\therefore (x, y) = \left(\frac{22}{2}, \frac{36}{2}\right)$$

$\therefore$ $(x, y) = (11, 18)$

Ans. The coordinates of the midpoint are **(11, 18).**

(v) Solution :



In the figure,

Seg AB represents the Church,

C is the position of the observer.

$\angle ACB$ is the angle of elevation.

$BC = 80$ m, $\angle ACB = 45°$

In right angled $\triangle ABC$,

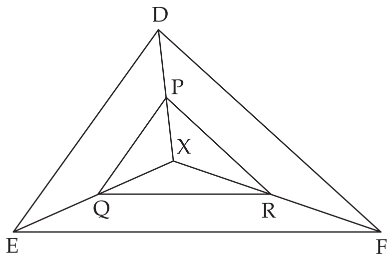
$\tan 45° = \frac{AB}{BC}$ ... (By definition)

$\therefore 1 = \frac{AB}{80}$ ... ($\tan 45° = 1$)

$\therefore AB = 80$ m

Ans. Height of the Church is **80 m.**

Q. 3. (A) (i)



Activity :

In $\triangle XDE$,

$PQ \parallel DE$ ... (Given)

$\therefore \frac{XP}{PD} = \frac{\boxed{XQ}}{QE}$ ... (Basic Proportionality theorem) ... (I)

In $\triangle XEF$,

$QR \parallel EF$ ... (Given)

$\therefore \frac{XQ}{\boxed{QE}} = \frac{XR}{\boxed{RF}}$ ... (**Basic Proportionality theorem**) ... (II)

$\therefore \frac{XP}{PD} = \frac{\boxed{XR}}{\boxed{RF}}$ ... [From (I) and (II)]

$\therefore$ seg PR $\parallel$ seg DF ... (By converse of basic proportionality theorem)

(ii) Activity :

Slope of line $=\dfrac{y_2-y_1}{x_2-x_1}$

$\therefore$ slope of line AB $=\dfrac{2-1}{8-6}=\boxed{\mathbf{\dfrac{1}{2}}}$ ... (I)

$\therefore$ slope of line BC $=\dfrac{4-2}{9-8}=\boxed{\mathbf{2}}$ ... (II)

$\therefore$ slope of line CD $=\dfrac{3-4}{7-9}=\boxed{\mathbf{\dfrac{1}{2}}}$ ... (III)

$\therefore$ slope of line DA $=\dfrac{3-1}{7-6}=\boxed{\mathbf{2}}$ ... (IV)

$\therefore$ slope of line AB = **slope of line CD** ... [From (I) and (III)]

line AB $\parallel$ line CD

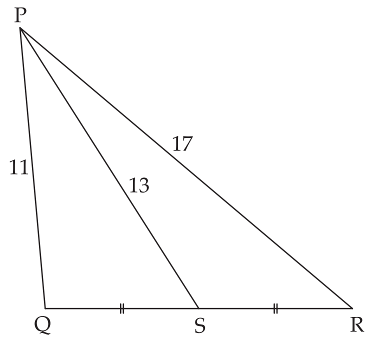
$\therefore$ slope of line BC = **slope of line DA** ... [From (II) and (IV)]

line BC $\parallel$ line DA

Both the pairs of opposite sides of the quadrilateral are parallel.

$\therefore \square$ ABCD is a parallelogram.

Q. 3. (B) (i) Solution :



In $\triangle PQR$,

seg PS is the median.

$\therefore$ by Apollonius theorem,

$PQ^2 + PR^2 = 2PS^2 + 2QS^2$

$\therefore 11^2 + 17^2 = 2(13)^2 + 2QS^2$

$\therefore 121 + 289 = 2(169) + 2QS^2$

$\therefore 410 = 338 + 2QS^2$ $\qquad \therefore 2QS^2 = 72$

$\therefore QS^2 = \dfrac{72}{2}$

$\therefore QS^2 = 36$

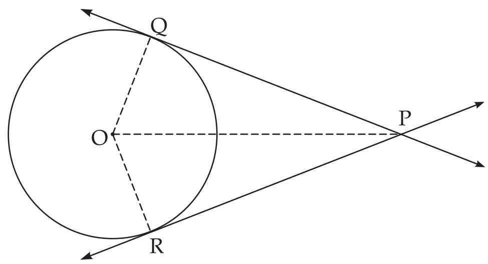
$\therefore QS = 6$ ... (By taking square roots)

$QR = 2QS$ ... (S is the midpoint of side QR)

$\therefore QR = 2 \times 6$

Ans. QR = **12.**

(ii)



Given : (1) A circle with centre O.

(2) lines PQ and PR are tangents to the circle at points Q and R respectively.

To prove : seg PQ $\cong$ seg PR

Construction : Draw seg OP, seg OQ and seg OR

Proof :

In $\triangle$OQP and $\triangle$ORP,

$\angle$OQP $=$ $\angle$ORP $= 90°$ ... (Tangent theorem)

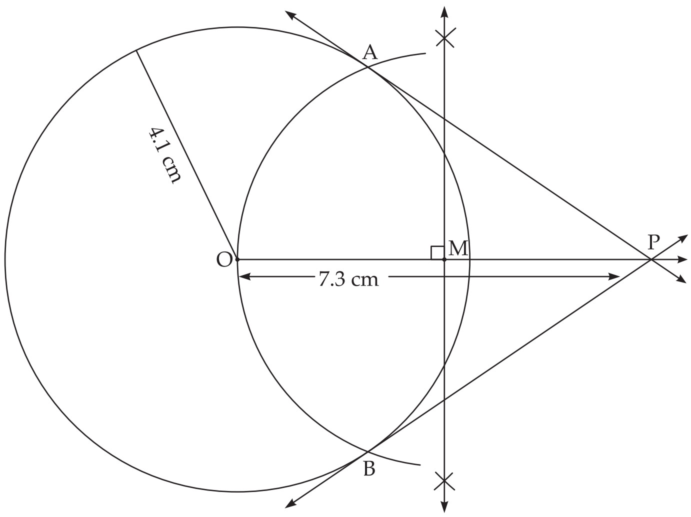
Hypotenuse OP $\cong$ Hypotenuse OP ... (Common side)

side OQ $\cong$ side OR ... (Radii of the same circle)

$\therefore$ $\triangle$OQP $\cong$ $\triangle$ORP ... (Hypotenuse side test)

$\therefore$ seg PQ $\cong$ seg PR ... (c.s.c.t.)

(iii) Ans.



Steps of construction :

(1) Draw circle with radius 4.1 cm and locate a point P at a distance of 7.3 cm.

(2) Draw perpendicular bisector of seg OP.

(3) Draw an arc to locate points of contact A and B.

(4) Draw tangents at points A and B.

(iv) Solution :

The length of cuboid $(l) = 16$ cm

its breadth $(b) = 11$ cm

its height $(h) = 10$ cm

The diameter of coin $= 2$ cm

$\therefore$ its radius $(r) = \frac{2}{2} = 1$ cm

its thickness $(h_1) = 2$ mm $= \frac{2}{10}$ cm

Number of coins that can be made $= \frac{\text{volume of cuboid}}{\text{volume of each coin}}$

$$= \frac{l \times b \times h}{\pi r^2 h_1}$$

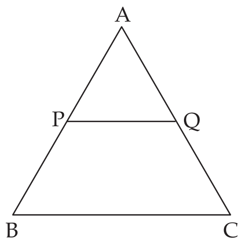
$$= \frac{16 \times 11 \times 10}{\frac{22}{7} \times 1 \times 1 \times \frac{2}{10}}$$

$$= \frac{16 \times 11 \times 10 \times 7 \times 10}{22 \times 1 \times 1 \times 2}$$

$$= 2800$$

Ans. 2800 coins can be made from the given metal cuboid.

Q. 4. (i) Solution :



Seg PQ divides $\triangle ABC$ into two parts having equal areas. ... (Given)

$\therefore A(\triangle APQ) = A(\square\ BPQC) = \frac{1}{2} A(\triangle ABC)$

$\therefore \frac{A(\triangle APQ)}{A(\triangle ABC)} = \frac{1}{2}$ ... (1)

In $\triangle APQ$ and $\triangle ABC$,

$\angle APQ \cong \angle ABC$ ... (Corresponding angles)

$\angle PAQ \cong \angle BAC$ ... (Common angle)

$\therefore \triangle APQ \sim \triangle ABC$ ... (AA test of similarity)

$\therefore \frac{A\triangle(APQ)}{A(\triangle ABC)} = \frac{AP^2}{AB^2}$ ... (Theorem of areas of similar triangles)

$\therefore \frac{1}{2} = \frac{AP^2}{AB^2}$ ... [From (1)]

$\therefore \frac{AP}{AB} = \frac{1}{\sqrt{2}}$ ... (Taking square roots)

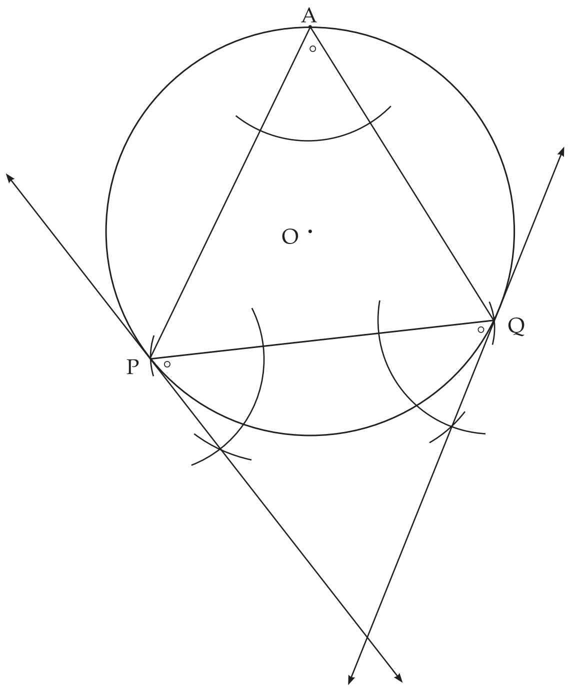
$$\therefore \frac{AB - BP}{AB} = \frac{1}{\sqrt{2}} \qquad \text{... (A-P-B)}$$

$$\therefore \frac{AB}{AB} - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$$

$$\therefore 1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}} \qquad \therefore \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$$

Ans. $\frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$

(ii) Ans.



Steps of construction :

(1) Draw a circle of radius 2.7 cm and chord PQ of length 4.5 cm.

(2) Draw $\angle PAQ$.

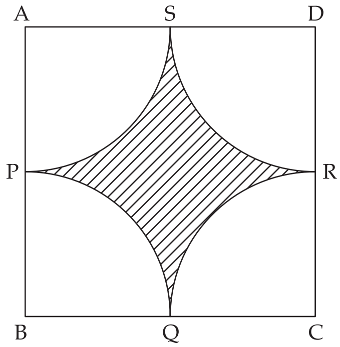
(3) Construct congruent angle to $\angle PAQ$ at P.

(4) Construct congruent angle to $\angle PAQ$ at Q.

(5) Draw tangent at P.

(6) Draw tangent at Q.

(iii) Solution :



$$A(\text{square}) = \text{side}^2 = 50^2 = 2500 \text{ m}^2$$

The radii of sectors A–SP, D-SR, C–RQ, B-PQ) $= \frac{1}{2} \times 50 = 25$ m

[$\because$ P, Q, R and S are midpoints of side AB, side BC, side CD and side AD respectively.]

$\therefore r = 25$ m

The sectors A–SP, D–SR, C–RQ, B–PQ are congruent sectors.

Angular measure of arc SP = angular measure of arc SR = angular measure of arc RQ = angular measure of arc PQ $= (\theta) = 90°$. ... (Angles of a square)

$A(\text{sector A–SP}) = \frac{\theta}{360} \times \pi r^2$ ... (Formula)

$= \frac{90}{360} \times \frac{22}{7} \times 25 \times 25$ ... (Substituting the values)

$= \left(\frac{1}{4} \times \frac{22}{7} \times 25 \times 25\right) \text{m}^2$

$A(\text{shaded region}) = A(\text{square}) - 4 \times A(\text{sector A–SP})$

$= 2500 - 4 \times \frac{1}{4} \times \frac{22}{7} \times 25 \times 25$

$= 2500 - \frac{13750}{7}$

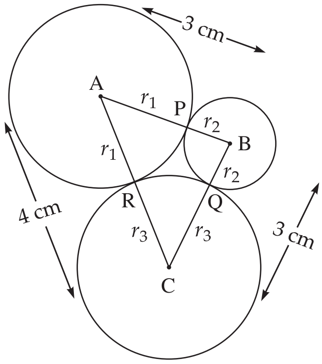
$= \frac{17500 - 13750}{7}$

$= \frac{3750}{7}$

$\approx 535.71 \text{ m}^2$

Ans. Area of the shaded region is **535.71 m²**.

Q. 5. (i) Solution :



Let the circles with centres A, B and C touch each other pairwise externally at points P, Q and R as shown.

A–P–B
B–Q–C } ... (By theorem of touching circles)
A–R–C

Let $AP = AR = r_1$
$BP = BQ = r_2$ } ... (Radii of the same circle)
$CQ = CR = r_3$

$AP + PB = AB$ ... (A-P-B)

$\therefore r_1 + r_2 = 3$ ... (1)

Similarly,

$r_2 + r_3 = 3$ ... (2)

$r_1 + r_3 = 4$ ... (3)

Adding (1), (2) and (3), we get,

$r_1 + r_2 + r_2 + r_3 + r_1 + r_3 = 3 + 3 + 4$

$\therefore 2r_1 + 2r_2 + 2r_3 = 10$

$\therefore 2(r_1 + r_2 + r_3) = 10$

$\therefore r_1 + r_2 + r_3 = 5$ ... (4)

Substituting (1) in (4),

$3 + r_3 = 5$

$\therefore r_3 = 5 - 3$

$\therefore r_3 = 2$

The radius of the circle with **centre C is 2 cm**.

Substituting (2) in (4),

$r_1 + 3 = 5$

$\therefore r_1 = 5 - 3$

$\therefore r_1 = 2$

The radius of the circle with **centre A is 2 cm**.

Substituting (3) in (4),

$r_2 + 4 = 5$

$\therefore r_2 = 5 - 4$

$\therefore r_2 = 1$

The radius of the circle with **centre B is 1 cm**.

(ii) Proof : $\sin\ \theta + \sin^2\theta = 1$ ... (Given) ... (1)

$\therefore \sin\ \theta = 1 - \sin^2\theta$

$\therefore \sin\ \theta = \cos^2\theta$ ... (2) ... $[\sin^2\theta + \cos^2\theta = 1 \quad \therefore \cos^2\theta = 1 - \sin^2\theta]$

Now, $\cos^2\theta + \cos^4\theta$

$= \cos^2\theta + (\cos^2\theta)^2$

$= \sin\theta + (\sin\theta)^2$ ... [From (2)]

$= \sin\theta + \sin^2\theta$

$= 1$ ... [From (1)]

$\therefore \cos^2\theta + \cos^4\theta = 1$

MATHEMATICS (PART – II)

BOARD'S QUESTION PAPER (NOVEMBER 2020)

(With Full Solution)

Time : 2 Hours] **[Total Marks : 40**

Note : *(i)* ***All*** *questions are compulsory.*

(ii) Use of calculator is ***not*** *allowed.*

(iii) The numbers to the right of the questions indicate full marks.

(iv) In case of MCQ's [Q. No. ***1(A)****], only the first attempt will be evaluated and will be given credit.*

(v) For every MCQ, the correct alternative (A), (B), (C) or (D) with subquestion number is to be written as an answer.

(vi) Draw proper figures for answers wherever necessary.

(vii) The marks of construction should be clear. Do not erase them.

(viii) Diagram is essential for writing the proof of the theorem.

Q. 1. (A) For each of the following subquestions four alternative answers are given. Choose the correct alternative and write its alphabet : **4**

(i) $\triangle ABC \sim \triangle PQR$ and $\angle A = 45°$, $\angle Q = 87°$, then $\angle C =$ ....

(A) 45° (B) 87° (C) 48° (D) 90°

(ii) $\angle PRQ$ is inscribed in the arc PRQ of a circle with centre 'O'. If $\angle PRQ = 75°$, then m(arc PRQ) = ...

(A) 75° (B) 150° (C) 285° (D) 210°

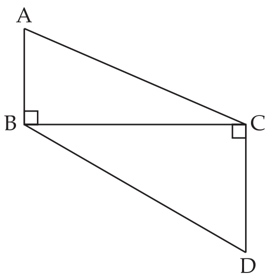
(iii) A line makes an angle of 60° with the positive direction of X-axis, so the slope of a line is ....

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

(iv) Radius of a sector of a circle is 5 cm and length of arc is 10 cm, then the area of a sector is ....

(A) 50 cm^2 (B) 25 cm^2 (C) 25 m^2 (D) 10 cm^2

Q. 1. (B) Solve the following subquestions : **4**

(i) 

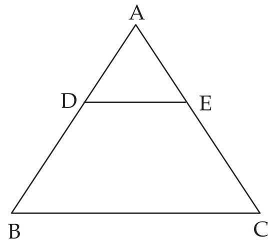
In the figure, seg AB $\perp$ seg BC and seg DC $\perp$ seg BC. If AB = 3 cm and CD = 4 cm, then find $\frac{A(\triangle ABC)}{A(\triangle DCB)}$.

(ii) In cyclic $\square$ABCD, $\angle B = 75°$, then find $\angle D$.

(iii) Point A, B, C are collinear. If slope of line AB is $-\frac{1}{2}$, then find the slope of line BC.

(iv) If $3 \sin \theta = 4 \cos \theta$, then find the value of $\tan \theta$.

Q. 2. (A) Complete the following activities and rewrite it : (*Any two*) **4**

(i) 

In $\triangle$ABC, seg DE$\parallel$side BC. If AD = 6 cm, DB = 9 cm, EC = 7.5 cm, then complete the following activity to find AE.

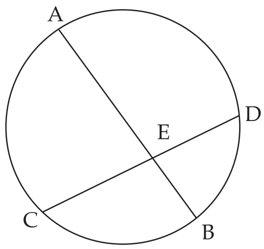
Activity : In $\triangle$ABC, seg DE$\parallel$side BC ... (given)

$\frac{AD}{DB} = \frac{AE}{EC}$ ... $\boxed{}$

$\therefore \frac{6}{9} = \frac{AE}{\boxed{}}$

$\therefore AE = \frac{6 \times 7.5}{\boxed{}}$

$\therefore AE = \boxed{}$

(ii) 

In the above figure, chord AB and chord CD intersect each other at point E. If AE = 15, EB = 6, CE = 12, then complete the activity to find ED.

Activity :

Chord AB and chord CD intersect each other at point E ... (Given)

$CE \times ED = AE \times EB$ ... $\boxed{}$

$\therefore \boxed{} \times ED = 15 \times 6$

$\therefore ED = \frac{\boxed{}}{12}$

$\therefore ED = \boxed{}$

(iii) If C(3, 5) and D(− 2, 3), then complete the following activity to find the distance between points C and D.

Activity :

Let $C(3, 5) \equiv (x_1, y_1)$, $D(-2, -3) \equiv (x_2, y_2)$

$CD = \sqrt{(x_2 - \boxed{})^2 + (y_2 - y_1)^2}$ ... (Formula)

$\therefore CD = \sqrt{(-2 - \boxed{})^2 + (-3 - 5)^2}$

$\therefore\ CD = \sqrt{\boxed{} + 64}$

$\therefore\ CD = \sqrt{\boxed{}}$

Q. 2. (B) Solve the following subquestions : *(Any four)* **8**

(i) $\triangle ABC \sim \triangle PQR$, $A(\triangle ABC) = 81\ cm^2$, $A(\triangle PQR) = 121\ cm^2$. If $BC = 6.3$ cm, then find QR.

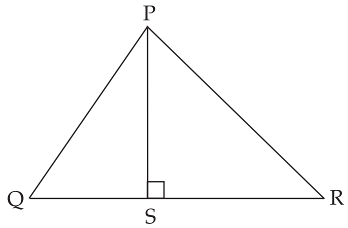
(ii) In $1\triangle PQR$, $\angle P = 60°$, $\angle Q = 90°$ and $QR = 6\sqrt{3}$ cm, then find the values of PR and PQ.

(iii) Find the slope of a line passing through the points A(2, 5) and B(4, -1).

(iv) Draw a circle with centre 'O' and radius 3.2 cm. Draw a tangent to the circle at any point P on it.

(v) Find the surface area of a sphere of radius 7 cm.

Q. 3. (A) Complete the following activities and rewrite it : (*Any one)* **3**

(i) 

In $\triangle PQR$, seg $PS \perp$ side QR, then complete the activity to prove $PQ^2 + RS^2 = PR^2 + QS^2$.

Activity :

In $\triangle PSQ$, $\angle PSQ = 90°$

$PS^2 + QS^2 = PQ^2$ ... (Pythagoras theorem)

$\therefore\ PS^2 = PQ^2 - \boxed{}$ ... (I)

Similarly,

In $\triangle PSR$, $\angle PSR = 90°$

$PS^2 + \boxed{} = PR^2$ ... (Pythagoras theorem)

$\therefore\ PS^2 = PR^2 - \boxed{}$ ... (II)

$\therefore\ PQ^2 - \boxed{} = \boxed{} - RS^2$ ... form (I) and (II)

$\therefore\ PQ^2 + \boxed{} = PR^2 + QS^2$

(ii) Measure of arc of a circle is 36° and its length is 176 cm. Then complete the following activity to find the radius of circle.

Activity :

Here, measure of arc $= \theta = 36°$

Length of arc $= l = 176$ cm

Length of arc $= (l) = \dfrac{\theta}{360} \times \boxed{}$ ... (formula)

$\therefore \square = \frac{36}{360} \times 2 \times \frac{22}{7} \times r$

$\therefore 176 = \frac{1}{\square} \times \frac{44}{7} \times r$

$\therefore r = \frac{176 \times \square}{44}$

$\therefore r = \square \times 70$

Radius of circle $(r) = \square$ cm

Q. 3. (B) Solve the following subquestions *(Any two)* : **6**

(i) Prove that, "The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines."

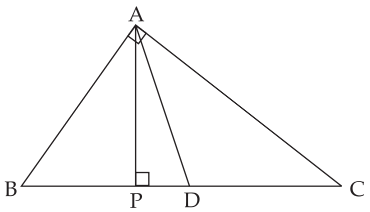
(ii) Draw a circle with centre 'O' and radius 3.4 cm. Draw a chord MN of length 5.7 cm in it. Construct tangents at points M and N to the circle.

(iii) Prove that : $\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$.

(iv) Radii of the top and base of frustum are 14 cm and 8 cm respectively. Its height is 8 cm. Find its curved surface area. ($\pi = 3.14$)

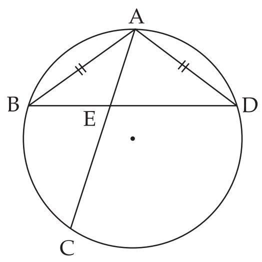
Q. 4. Solve the following subquestions *(Any two)* : **8**

(i)



In $\triangle ABC$, $\angle BAC = 90°$, seg $AP \perp$ side BC, B-P-C. Point D is the mid-point of side BC, then prove that $2AD^2 = BD^2 + CD^2$.

(ii)



In the above figure, chord $AB \cong$ chord AD. Chord AC and chord BD intersect each other at point E. Then prove that :

$AB^2 = AE \times AC$.

(iii) A straight road leads to the foot of the tower of height 48 m. From the top of the tower the angles of depression of two cars standing on the road are 30° and 60° respectively. Find the distance between the two cars. ($\sqrt{3} = 1.73$)

Q. 5. Solve the following subquestions *(Any one)* : **3**

(i) Let M be a point of contact of two internally touching circles. Let line AMB be their common tangent. The chord CD of the bigger circle touches the smaller circle at point N. The chord CM and chord DM of bigger circle intersect the smaller circle at point P and R respectively.

(a) From the above information draw the suitable figure.

(b) Draw seg NR and seg NM and write the two pairs of congruent angles in smaller circle considering tangent and chord.

(c) By using the property which is use in *(b)* write the two pairs of congruent angles in the bigger circle.

(ii) Draw a circle with centre 'O' and radius 3 cm. Draw a tangent segment PA having length $\sqrt{40}$ cm from an exterior point P.

SOLUTION : BOARD'S QUESTION PAPER (NOVEMBER 2020)

Q. 1. (A) (i) (C)

(ii) (D)

(iii) (C)

(iv) (B)

Hints : Only for guidance. Students are not expected to write this.

(i) (C) For similar triangle corresponding angles are congruent. The sum of the angles of a triangle $= 180°$.

(ii) (D) m(minor arc PQ) $= 2m\angle PRQ = 2 \times 75° = 150°$

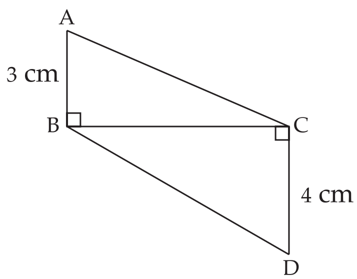
m(arc PRQ) $+ m$(minor arc PQ) $= m$(circle) $= 360°$

$\therefore$ m(arc PRQ) $= 360° - 150° = 210°$.

(iii) (C) The slope $= \tan\theta$. $\tan 60° = \sqrt{3}$.

(iv) (B) Area of the sector $= \dfrac{l(\text{arc}) \times \text{radius}}{2}$

Q. 1. (B) (i)



$\triangle ABC$ and $\triangle DCB$ have same base BC.

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle DCB)} = \dfrac{AB}{DC}$ ... (Triangles with equal base)

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle DCB)} = \mathbf{\dfrac{3}{4}}$

(ii) $\square$ABCD is cyclic.

$\angle B + \angle D = 180°$ ... (Opposite angles of cyclic quadrilateral are supplementary)

$\therefore 75° + \angle D = 180°$

$\therefore \angle D = 180° - 75°$

$\therefore \angle D =$ **105°**

(iii) Slope of line AB $= -\dfrac{1}{2}$

Slope of line BC = Slope of line AB

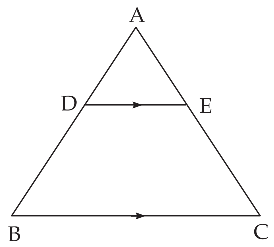
$\therefore$ slope of line BC $= \mathbf{-\dfrac{1}{2}}$

(iv) $3 \sin \theta = 4 \cos \theta$ ... (Given)

$\therefore \dfrac{\sin \theta}{\cos \theta} = \dfrac{4}{3}$

$\therefore \tan \theta = \mathbf{\dfrac{4}{3}}$

Q. 2. (A) (i)



Activity :

In $\triangle$ABC, seg DE $\parallel$ side BC ... (Given)

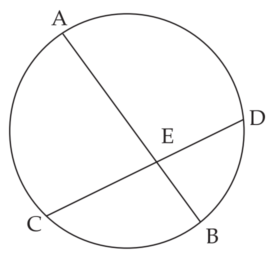
$\dfrac{AD}{DB} = \dfrac{AE}{EC}$ ... $\boxed{\textbf{(Basic Proportionality Theorem)}}$

$\therefore \dfrac{6}{9} = \dfrac{AE}{\boxed{\mathbf{7.5}}}$

$\therefore AE = \dfrac{6 \times 7.5}{\boxed{\mathbf{9}}}$

$\therefore AE = \boxed{\mathbf{5}}$

(ii)



Activity :

Chord AB and chord CD intersect each other at point E. ... (Given)

$CE \times ED = AE \times EB$ ... $\boxed{\textbf{(Theorem of internal division of chords)}}$

$\therefore \boxed{\mathbf{12}} \times ED = 15 \times 6$

$\therefore ED = \dfrac{\boxed{\mathbf{90}}}{12}$

$\therefore ED = \boxed{\mathbf{7.5}}$

(iii) Activity :

Let $C(3, 5) \equiv (x_1, y_1)$

$D(-2, -3) \equiv (x_2, y_2)$

$CD = \sqrt{(x_2 - \boxed{\mathbf{x_1}})^2 + (y_2 - y_1)^2}$

$\therefore\ CD = \sqrt{(-2 - \boxed{3})^2 + (-3-5)^2}$

$\therefore\ CD = \sqrt{\boxed{25} + 64}$

$\therefore\ CD = \sqrt{\boxed{89}}$

Q. 2. (B) (i) Solution :

$\triangle ABC \sim \triangle PQR$ ... (Given)

$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{BC^2}{QR^2}$ ... (Theorem of areas of similar triangles)

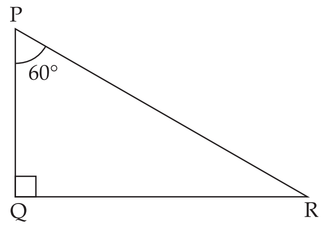
$\therefore\ \frac{81}{121} = \frac{(6.3)^2}{QR^2}$

$\therefore\ \frac{9}{11} = \frac{6.3}{QR}$ ... (On taking square roots of both the sides)

$\therefore\ QR = \frac{11 \times 6.3}{9}$

$\therefore\ QR = 7.7$ cm **Ans.** QR = **7.7 cm**

(ii) Solution :



In $\triangle PQR$,

$\angle P = 60°$, $\angle Q = 90°$,

$\therefore\ \angle R = 30°$ ... (Remaining angle of a triangle)

$\therefore\ \triangle PQR$ is a $30° - 60° - 90°$ triangle.

$\therefore$ By $30° - 60° - 90°$ triangle theorem,

$QR = \frac{\sqrt{3}}{2} \times PR$ ... (Side opposite to 30°)

$\therefore\ 6\sqrt{3} = \frac{\sqrt{3}}{2} \times PR$

$\therefore\ PR = \frac{6\sqrt{3} \times 2}{\sqrt{3}}$

$\therefore\ PR = 12$ cm

$PQ = \frac{1}{2} \times PR$ ... (Side opposite to 30°)

$PQ = \frac{1}{2} \times 12$

$\therefore\ PQ = 6$ cm **Ans.** PR = **12 cm** and PQ = **6 cm.**

(iii) Solution :

Let A (2, 5) $\equiv (x_1, y_1)$

B (4, -1) $\equiv (x_2, y_2)$

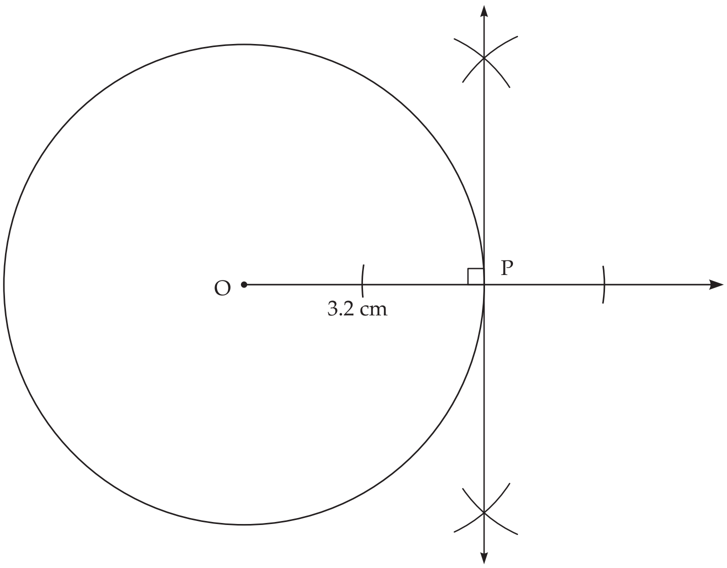
$$\text{Slope of line AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1-5}{4-2}$$

$$= -\frac{6}{2} = -3$$

Ans. Slope of line AB = $-$**3.**

(iv) Ans.



(a) Draw a circle with given radius and point P on it.

(b) Draw a tangent at point P.

(v) Solution :

The radius of the sphere $(r) = 7$ cm

$$\text{Surface area of the sphere} = 4\pi r^2$$

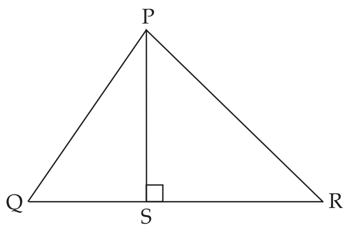
$$= 4 \times \frac{22}{7} \times 7 \times 7$$

$$= 88 \times 7$$

$$= 616 \text{ sq. cm.}$$

Ans. Surface area of the sphere is **616 sq. cm.**

Q. 3. (A) (i)



Activity :

In $\triangle PSQ$,

$\angle PSQ = 90°$

$PS^2 + QS^2 = PQ^2$ ... (Pythagoras theorem)

$\therefore PS^2 = PQ^2 - \boxed{\mathbf{QS^2}}$ ... (1)

Similarly, in $\triangle PSR$, $\angle PSR = 90°$

$PS^2 + \boxed{\mathbf{RS^2}} = PR^2$ ... (Pythagoras theorem)

$\therefore PS^2 = PR^2 - \boxed{\mathbf{RS^2}}$ ... (2)

$\therefore PQ^2 - \boxed{\mathbf{QS^2}} = \boxed{\mathbf{PR^2}} - RS^2$ ... [from (1) and (2)]

$\therefore PQ^2 + \boxed{\mathbf{RS^2}} = PR^2 + QS^2$

(ii) Activity :

Here, measure of arc $= \theta = 36°$

Length of arc $= l = 176$ cm

Length of arc $(l) = \frac{\theta}{360} \times \boxed{\mathbf{2\pi r}}$ ... (Formula)

$\therefore \boxed{\mathbf{176}} = \frac{36}{360} \times 2 \times \frac{22}{7} \times r$

$\therefore 176 = \frac{1}{\boxed{\mathbf{10}}} \times \frac{44}{7} \times r$

$\therefore r = \frac{176 \times \boxed{\mathbf{70}}}{44}$

$\therefore r = \boxed{\mathbf{4}} \times 70$

$\therefore$ Radius of the circle $(r) = \boxed{\mathbf{280}}$ cm

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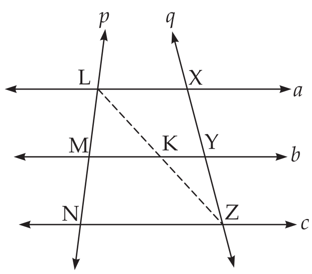
Q. 3. (B) (i)



Given : Line $a \parallel$ line $b \parallel$ line c. Transversal p intersects lines a, b and c in points L, M and N respectively. Transversal q intersects lines a, b and c in points X, Y and Z respectively.

To prove : $\frac{LM}{MN} = \frac{XY}{YZ}$.

Construction : Draw seg LZ. Let seg LZ intersect seg MY at point K such that M-K-Y and L-K-Z.

Proof : In $\triangle LNZ$,

seg MK $\parallel$ side NZ ... (Given)

$\therefore$ by basic proportionality theorem,

$$\frac{LM}{MN} = \frac{LK}{KZ} \quad \text{... (1)}$$

In $\triangle LZX$

seg KY $\parallel$ side LX ... (Given)

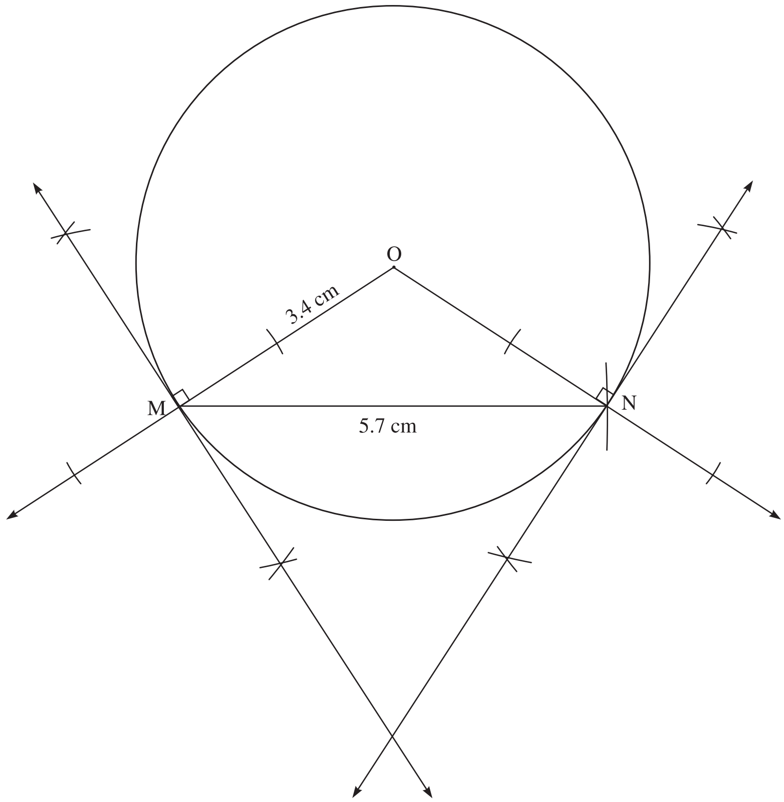
$\therefore$ by basic proportionality theorem,

$$\frac{LK}{KZ} = \frac{XY}{YZ} \quad \text{... (2)}$$

$\therefore$ from (1) and (2),

$$\frac{LM}{MN} = \frac{XY}{YZ}.$$

(ii) Ans.



(a) Draw a circle with centre O and radius 3.4 cm.

(b) Draw chord MN of length 5.7 cm.

(c) Draw tangent through points M and N.

(iii) Proof :

$$\text{LHS} = \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{(\sec\theta - \tan\theta)} \times \frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{\sec\theta + \tan\theta}{1} \qquad \dots \left(\begin{array}{l} \sec^2\theta = 1 + \tan^2\theta \\ \therefore \sec^2\theta - \tan^2\theta = 1 \end{array} \right)$$

$= \text{RHS}$

$\therefore$ LHS = RHS

(iv) Solution :

For frustum,

Radii of the top $(r_1) = 14$ cm,

radii of the bottom $(r_2) = 8$ cm,

height $(h) = 8$ cm

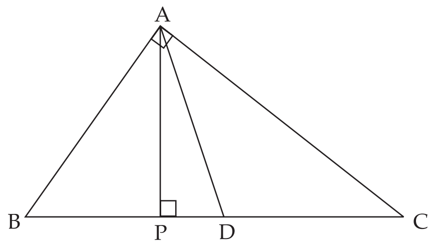
$$\text{Slant height of the frustum } (l) = \sqrt{h^2 + (r_1 - r_2)^2}$$
$$= \sqrt{8^2 + (14 - 8)^2}$$
$$= \sqrt{64 + 36}$$

Slant height $(l) = 10$ cm

$$\text{Curved surface area of frustum} = \pi(r_1 + r_2)l$$
$$= 3.14\ (14 + 8) \times 10$$
$$= 3.14 \times 220$$
$$= 690.8 \text{ cm}^2$$

Ans. Curved surface area of frustum is **690.8 cm²**.

Q. 4. (i)



Proof :

In $\triangle$ABC,

$\angle BAC = 90°$ ... (Given)

$AB^2 + AC^2 = BC^2$ ... (1) ... (Pythagoras theorem)

In $\triangle$ABC,

D is the midpoint of BC.

$AB^2 + AC^2 = 2AD^2 + 2BD^2$ ... (Appollonius theorem)

$\therefore 2AD^2 = AB^2 + AC^2 - 2BD^2$

$\therefore 2AD^2 = BC^2 - 2BD^2$ ... [from (1)]

$\therefore 2AD^2 = (2BD)^2 - 2BD^2$ ... ($\because$ BC = 2BD)

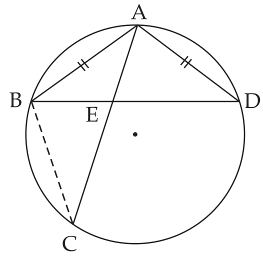
$\therefore 2AD^2 = 4BD^2 - 2BD^2$

$\therefore 2AD^2 = 2BD^2$

$\therefore 2AD^2 = BD^2 + BD^2$

$\therefore 2AD^2 = BD^2 + CD^2$ ... ($\because$ BD = CD)

(ii)



Proof : Draw seg BC.

In $\triangle ABD$,

seg AB $\cong$ seg AD ... (Given)

$\therefore \angle ABD \cong \angle ADB$ ... (1) ... (Isosceles triangle theorem)

$\therefore \angle ACB \cong \angle ADB$ ... (2) ... (Angles inscribed in the same arc)

$\therefore \angle ABD \cong \angle ACB$ ... [From (1) and (2)]

i.e. $\angle ABE \cong \angle ACB$ ... (B-E-D) ... (3)

In $\triangle ABE$ and $\triangle ACB$

$\angle ABE \cong \angle ACB$ ... [From (3)]

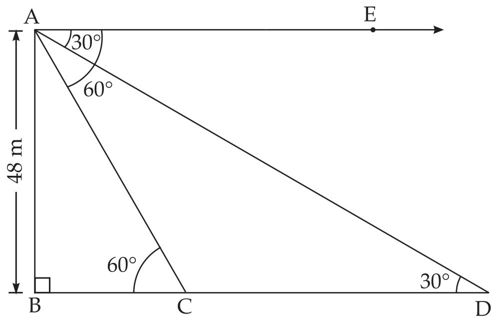
$\angle BAE \cong \angle BAC$ ... (Common angle)

$\therefore \triangle ABE \sim \triangle ACB$ ... (AA test of similarity)

$\therefore \dfrac{AB}{AC} = \dfrac{AE}{AB}$ ... (Corresponding sides of similar triangles)

$\therefore AB^2 = AE \times AC$

(iii) Solution :



AB represents the tower.

$\therefore AB = 48$ m

Let C and D be the position of two cars.

$\angle EAD$ and $\angle EAC$ are the angles of depression.

$\angle EAD = 30°$ and $\angle EAC = 60°$

Now, $\angle EAD = \angle ADB = 30°$
$\angle EAC = \angle ACB = 60°$ } ... (Alternate angles)

In right angled $\triangle ABC$,

$\tan 60° = \dfrac{AB}{BC}$

$\therefore \sqrt{3} = \dfrac{48}{BC}$

$\therefore BC = \dfrac{48}{\sqrt{3}}$

$\therefore BC = \dfrac{48}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$\therefore BC = \dfrac{48\sqrt{3}}{3}$

$\therefore BC = 16\sqrt{3}$ m

Similarly, in right angled $\triangle ABD$,

$\tan 30° = \dfrac{AB}{BD}$

$\therefore \dfrac{1}{\sqrt{3}} = \dfrac{48}{BD}$

$\therefore BD = 48\sqrt{3}$ m

Distance between two cars = CD = BD − BC

$\therefore CD = 48\sqrt{3} - 16\sqrt{3}$

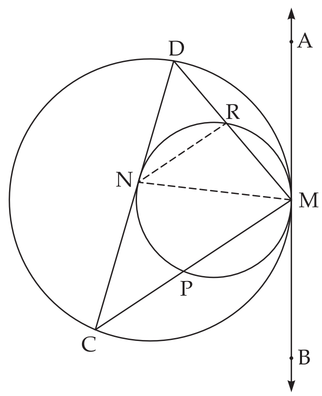
$\therefore CD = 32\sqrt{3}$

$\therefore CD = 32 \times 1.73$

$\therefore CD = 55.36$ m

Ans. The distance between two cars is **55.36 m.**

Q. 5. (i) Ans. (a)



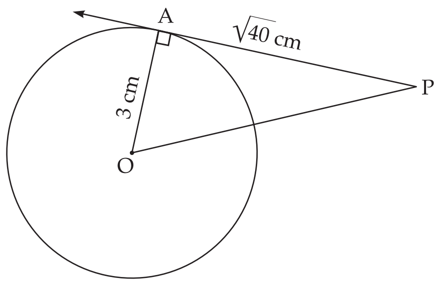
(b) $\angle DNR \cong \angle NMR$

$\angle RNM \cong \angle RMA$

(c) $\angle DCM \cong \angle DMA$

$\angle CDM \cong \angle CMB$

(ii) Analysis :



In $\triangle OAP$,

$\angle OAP = 90°$ ... (Tangent theorem)

$OP^2 = OA^2 + PA^2$ ... (Pythagoras theorem)

$\therefore OP^2 = 3^2 + (\sqrt{40})^2$

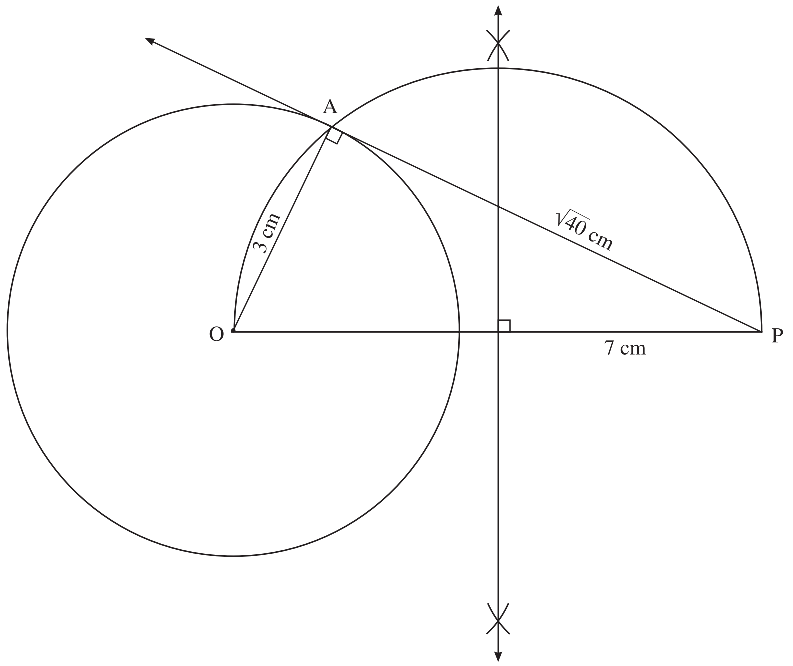
$\therefore OP^2 = 9 + 40$

$\therefore OP^2 = 49$

$\therefore OP = 7$ cm ... (Taking square roots)

$\therefore$ Distance of point P from centre O is 7 cm

Ans.



Steps of construction :

(1) Draw a circle of given radius and seg OP.

(2) Draw perpendicular bisector of seg OP.

(3) Draw a semicircle with diameter OP.

(4) Draw tangent segment PA or ray PA.

* * *