

SOLUTION : PRACTICE QUESTION PAPER 4

- Q. 1. (A)** (i) (B)
 (ii) (B)
 (iii) (D)
 (iv) (A)

Q. 1. (A) Explanation to the answers to MCQs in this question has been given below for students' guidance. Please note that, **Students are not expected to write the explanation in the examination.**

Explanations :

- (i) (B) Areas of triangles with equal heights are proportional to their corresponding bases.

$$\therefore \frac{A(\triangle ABD)}{A(\triangle ADC)} = \frac{BD}{DC}$$

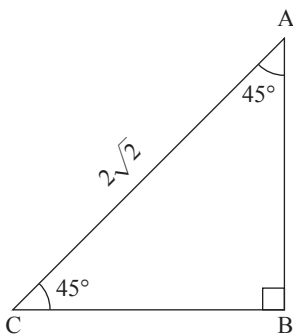
- (ii) (B) [In cyclic $\square ABCD$, $\angle A + \angle C = 180^\circ$, $2\angle A = 3\angle C$

$$\therefore 2\angle A - 3\angle C = 0. \text{ Solve two equations}]$$

- (iii) (D) [Slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$]

- (iv) (A) [Volume of cube = (side)³].

- Q. 1. (B)** (i) **Solution :**



$\triangle ABC$ is a $45^\circ - 45^\circ - 90^\circ$ triangle

\therefore by $45^\circ - 45^\circ - 90^\circ$ triangle theorem,

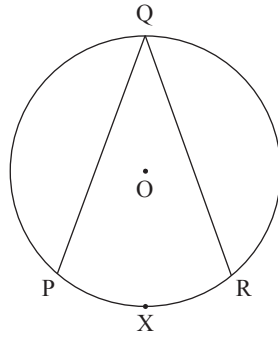
$$AB = \frac{1}{\sqrt{2}} AC \quad \dots \text{ (Side opposite to } 45^\circ \text{)}$$

$$\therefore AB = \frac{1}{\sqrt{2}} \times 2\sqrt{2}$$

$$\therefore AB = 2$$

Ans. $AB = 2$.

(ii) Solution :



$$m(\text{arc PXR}) = 80^\circ$$

$$\angle \text{PQR} = \frac{1}{2} \times m(\text{arc PXR}) \quad \dots \text{ [Inscribed angle theorem]}$$

$$= \frac{1}{2} \times 80^\circ = 40^\circ$$

Ans. Measure of $\angle \text{PQR}$ is 40° .

(iii) **Ans.**

The longest chord of the circle is 8 cm.

Diameter is the longest chord of the circle.

$$\text{Radius} = \frac{1}{2} \times \text{diameter}$$

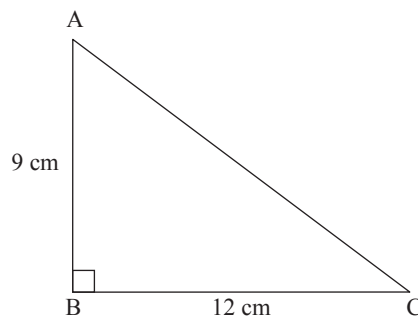
$$= \frac{1}{2} \times 8$$

$$= 4 \text{ cm}$$

The radius of the circle is 4 cm.

(iv) **Ans.** The point of intersection of line $x = 2$ and $y = -3$ is $(2, -3)$.

Q. 2. (A) (i)



Activity :

$\triangle \text{ABC}$ is a right angled triangle,

$\text{AB} = 9 \text{ cm}$, $\text{BC} = 12 \text{ cm}$

By Pythagoras theorem,

$$\begin{aligned}AC^2 &= AB^2 + BC^2 \\ &= 9^2 + \boxed{12^2} \\ &= 81 + \boxed{144} = \boxed{225}\end{aligned}$$

$$\therefore AC = \boxed{15} \text{ cm.}$$

(ii) Activity :

$$\begin{aligned}\text{LHS} &= \frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} \\ &= \frac{1 + \sin \theta + \boxed{(1 - \sin \theta)}}{(1 - \sin \theta)(1 + \sin \theta)} \\ &= \frac{1 + \sin \theta + 1 - \sin \theta}{1 - \boxed{\sin^2 \theta}} \\ &= \frac{2}{\boxed{\cos^2 \theta}} = 2 \times \frac{1}{\cos^2 \theta} \\ &= 2 \boxed{\sec^2 \theta} \\ &= \text{RHS.}\end{aligned}$$

(iii) Activity :

The radius of the sector (r) = 3.5 cm

Length of the arc = 2.2 cm

$$\begin{aligned}\text{Area of the sector} &= \frac{\text{Length of the arc} \times \boxed{\text{radius}}}{2} \\ &= \frac{\boxed{2.2} \times \boxed{3.5}}{2} \\ &= \boxed{3.85} \text{ cm}^2.\end{aligned}$$

Q. 2. (B) Solution :

(i) In $\triangle MNP$,

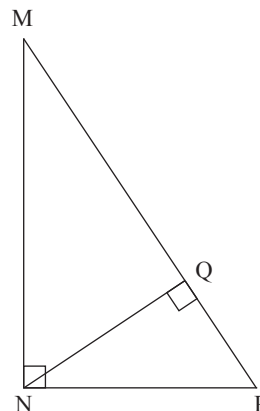
$$\angle MNP = 90^\circ$$

$NQ \perp MP$

\therefore by theorem of geometric mean,

$$NQ^2 = MQ \times QP$$

$$\therefore NQ^2 = 9 \times 4$$

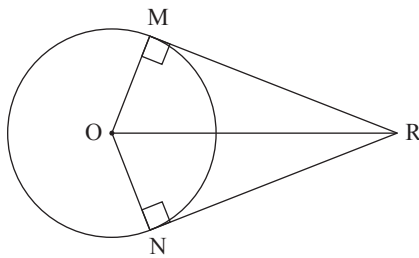


$$\therefore NQ^2 = 36$$

$$\therefore NQ = 6$$

Ans. $NQ = 6$.

(ii)



Proof : In $\triangle RMO$ and $\triangle RNO$

$$\angle RMO = \angle RNO = 90^\circ \quad \dots \text{ (Tangent theorem)}$$

$$\text{Hypotenuse } OR \cong \text{hypotenuse } OR \quad \dots \text{ (Common side)}$$

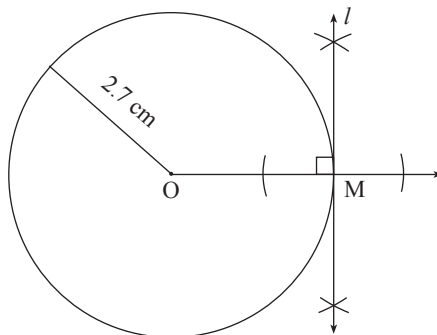
$$\text{Side } OM \cong \text{side } ON \quad \dots \text{ (Radii of the same circle)}$$

$$\therefore \triangle RMO \cong \triangle RNO \quad \dots \text{ (Hypotenuse side theorem)}$$

$$\left. \begin{array}{l} \therefore \angle MRO \cong \angle NRO \\ \text{and } \angle MOR \cong \angle NOR \end{array} \right\} \quad \dots \text{ (c.a.c.t.)}$$

\therefore seg OR bisects $\angle MRN$ and $\angle MON$.

(iii)



(iv) **Solution :** $P(-12, -3)$ and $Q(4, k)$.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$.

Here, $x_1 = -12$, $y_1 = -3$, $x_2 = 4$ and $y_2 = k$.

Slope of the line $= m = \frac{1}{2}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \frac{1}{2} = \frac{k - (-3)}{4 - (-12)}$$

$$\therefore \frac{1}{2} = \frac{k+3}{4+12}$$

$$\therefore \frac{1}{2} = \frac{k+3}{16}$$

$$\therefore 2(k+3) = 16$$

$$\therefore (k+3) = \frac{16}{2} \quad \therefore k+3 = 8$$

$$\therefore k = 8 - 3$$

$$\therefore k = 5$$

Ans. $k = 5$.

(v) Proof :

$$\text{LHS} = \frac{1}{\sec \theta - \tan \theta}$$

$$= \frac{1}{(\sec \theta - \tan \theta)} \times \frac{(\sec \theta + \tan \theta)}{(\sec \theta + \tan \theta)}$$

$$= \frac{1(\sec \theta + \tan \theta)}{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta)}$$

$$= \frac{\sec \theta + \tan \theta}{\sec^2 \theta - \tan^2 \theta}$$

$$= \frac{\sec \theta + \tan \theta}{1}$$

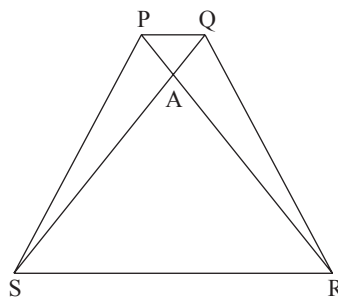
$$\dots \left[\begin{array}{l} \because 1 + \tan^2 \theta = \sec^2 \theta \\ \therefore \sec^2 \theta - \tan^2 \theta = 1 \end{array} \right]$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$\therefore \frac{1}{\sec \theta - \tan \theta} = \sec \theta + \tan \theta.$$

Q. 3. (A) (i)



Activity :

In $\triangle PQA$ and $\triangle RSA$,

$$\angle PQA \cong \angle RSA \quad \dots \quad \boxed{\text{(Alternate interior angles)}}$$

$$\angle PAQ \cong \angle RAS \quad \dots \quad \boxed{\text{(Vertically opposite angles)}}$$

$$\therefore \triangle PQA \sim \triangle RSA \quad \dots \quad \boxed{\text{AA test of similarity}}$$

$$\frac{PQ}{RS} = \frac{AP}{AR} \quad \dots \text{ (Corresponding sides of similar triangles) } \dots (1)$$

Substituting $AR = 5AP$ in (1)

$$\therefore \frac{PQ}{RS} = \frac{AP}{5AP}$$

$$\therefore \frac{PQ}{RS} = \frac{1}{5}$$

$$\therefore RS = 5PQ.$$

(ii) Activity :

P (2, -2), Q (7, 3), R (11, -1) and S (6, -6) are the given points.

Using distance formula,

$$\text{distance between two points} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \boxed{5\sqrt{2}} \quad \dots (1)$$

$$QR = \boxed{4\sqrt{2}} \quad \dots (2)$$

$$SR = \boxed{5\sqrt{2}} \quad \dots (3)$$

$$PS = \boxed{4\sqrt{2}} \quad \dots (4)$$

\therefore from (1), (2), (3) and (4)

$$PQ = \boxed{SR}$$

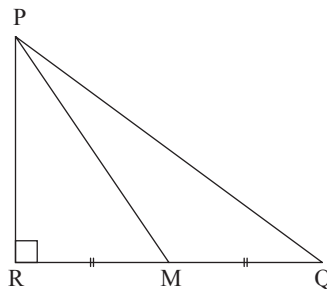
$$QR = \boxed{PS}$$

\therefore \square PQRS is a parallelogram

\dots [A quadrilateral is a parallelogram, if its opposite sides are equal]

\therefore P(2, -2), Q(7, 3), R(11, -1) and S(6, -6) are the vertices of a parallelogram.

Q. 3. (B) (i)



Proof : In $\triangle PRQ$,

$$\angle PRQ = 90^\circ \quad \dots \text{ (Given)}$$

\therefore by Pythagoras theorem,

$$PQ^2 = PR^2 + QR^2 \quad \dots (1)$$

In $\triangle PRM$,

$$\angle PRM = 90^\circ \quad \dots \text{ (Given)}$$

\therefore by Pythagoras theorem,

$$PM^2 = PR^2 + RM^2 \quad \dots (2)$$

$$RM = \frac{1}{2} RQ \quad \dots \text{ (M is the midpoint of seg RQ) } \dots (3)$$

$$\therefore PM^2 = PR^2 + \left(\frac{1}{2} RQ\right)^2 \quad \dots \text{ [From (2) and (3)]}$$

$$\therefore PM^2 = PR^2 + \frac{1}{4} RQ^2$$

Multiplying each term with 4, we get

$$4PM^2 = 4PR^2 + RQ^2$$

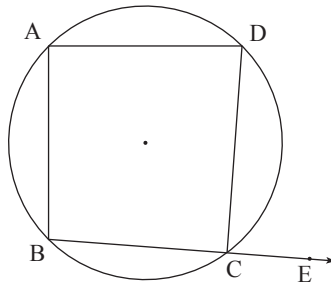
$$\therefore 4PM^2 = 3PR^2 + (PR^2 + RQ^2)$$

$$\therefore 4PM^2 = 3PR^2 + PQ^2 \quad \dots \text{ [From (1)]}$$

$$\therefore 4PM^2 - 3PR^2 = PQ^2$$

$$\text{i.e. } PQ^2 = 4PM^2 - 3PR^2.$$

(ii)



Given : $\square ABCD$ is cyclic. $\angle DCE$ is an exterior angle of $\square ABCD$.

To prove : $\angle DCE \cong \angle BAD$

Proof :

$$\angle DCE + \angle BCD = 180^\circ \quad \dots \text{ (Linear pair of angles) } \dots (1)$$

$\square ABCD$ is cyclic.

$$\therefore \angle BAD + \angle BCD = 180^\circ \quad \dots \text{ (Theorem of cyclic quadrilateral) } \dots (2)$$

\therefore from (1) and (2), we get

$$\angle DCE + \angle BCD = \angle BAD + \angle BCD$$

$$\therefore \angle DCE = \angle BAD$$

$$\therefore \angle DCE \cong \angle BAD.$$

(iii) Solution :

Let seg AB and seg CD represent the two buildings. Seg BD represents the road.

Seg AE \perp seg CD

... C-E-D.

\angle CAE is the angle of elevation.

AB = 10 m, BD = 12 m and \angle CAE = 60° .

\square ABDE is a rectangle

DE = AB = 10 m

AE = BD = 12 m ... (Opposite sides of rectangle are equal)

In right angled \triangle CEA,

$$\tan 60^\circ = \frac{CE}{AE} \quad \dots \text{ (By definition)}$$

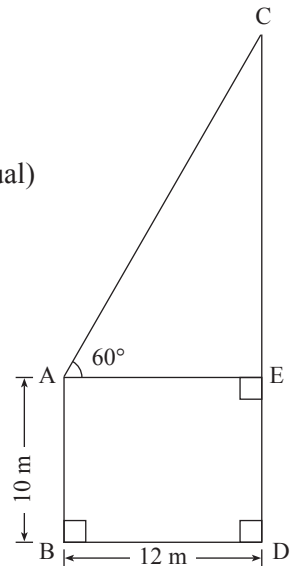
$$\therefore \sqrt{3} = \frac{CE}{12} \quad \dots \text{ (} \tan 60^\circ = \sqrt{3} \text{)}$$

$$\therefore CE = 12\sqrt{3} \text{ m}$$

$$CD = CE + ED \quad \dots \text{ (C-E-D)}$$

$$\therefore CD = (12\sqrt{3} + 10) \text{ m}$$

Ans. The height of second building is **$(12\sqrt{3} + 10)$ m.**



(iv) Solution : Area of the shaded region is 114 cm^2 .

The shaded region is segment PRQ.

$$A(\text{segment PRQ}) = 114 \text{ cm}^2$$

$$m(\text{arc PRQ}) = \angle \text{POQ} = \theta = 90^\circ$$

$$A(\text{segment PRQ}) = r^2 \left[\frac{\pi\theta}{360} - \frac{\sin \theta}{2} \right]$$

$$\therefore 114 = r^2 \left[\frac{3.14 \times 90}{360} - \frac{\sin 90^\circ}{2} \right]$$

$$\therefore 114 = r^2 \left[\frac{3.14}{4} - \frac{1}{2} \right]$$

$$\therefore 114 = r^2 \left[\frac{3.14 - 2}{4} \right]$$

$$\therefore 114 = r^2 \times \frac{1.14}{4}$$

$$\therefore r^2 = \frac{114 \times 4 \times 100}{1.14 \times 100}$$

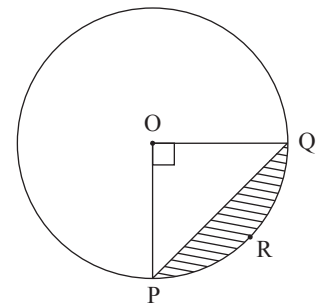
$$\therefore r^2 = \frac{114 \times 4 \times 100}{114}$$

$$\therefore r^2 = 400$$

$$\therefore r = \sqrt{400}$$

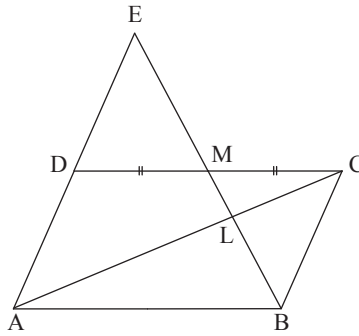
$$\therefore r = 20 \text{ cm}$$

Ans. The radius of the circle is **20 cm.**



$$\dots \text{ (} \sin 90^\circ = 1 \text{)}$$

Q. 4. (i)



Proof : Side AD \parallel side BC ... (Opposite sides of a parallelogram)

i.e. AE \parallel BC and BE is the transversal.

$$\therefore \angle BEA \cong \angle EBC \quad \dots \text{ (Alternate angles)}$$

i.e. $\angle MED \cong \angle MBC$

$$\angle DME \cong \angle BMC \quad \dots \text{ (Vertically opposite angles)}$$

$$\therefore \triangle EMD \sim \triangle BMC \quad \dots \text{ (AA test of similarity)}$$

$$\therefore \frac{MD}{MC} = \frac{ED}{BC} \quad \dots \text{ (Corresponding sides of similar triangles are proportional)}$$

$$MD = MC \quad \dots \text{ (M is the midpoint of DC)}$$

$$\therefore \frac{MD}{MC} = \frac{ED}{BC} = 1$$

$$\therefore ED = BC \quad \dots \text{ (1)}$$

$$BC = AD \quad \dots \text{ (Opposite sides of a parallelogram) ... (2)}$$

From (1) and (2), $ED = AD$

$$AD + ED = AD + AD \quad \dots \text{ (Adding AD to both the sides)}$$

$$\therefore AE = 2AD \quad \dots \text{ (A - D - E) ... (3)}$$

$$\therefore AE = 2BC \quad \dots \text{ (}\because AD = BC, \text{ opposite sides of parallelogram)}$$

AE \parallel BC and AC is the transversal.

$$\therefore \angle EAC \cong \angle BCA \quad \dots \text{ (Alternate angles)}$$

$$\angle ELA \cong \angle BLC \quad \dots \text{ (Vertically opposite angles)}$$

$$\therefore \triangle AEL \sim \triangle CBL \quad \dots \text{ (AA test of similarity)}$$

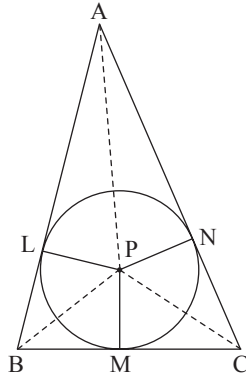
$$\therefore \frac{EL}{BL} = \frac{AE}{BC} \quad \dots \text{ (Corresponding sides of similar triangles are proportional)}$$

$$\therefore \frac{EL}{BL} = \frac{2BC}{BC} \quad \dots \text{ [From (3)]}$$

$$\therefore \frac{EL}{BL} = 2$$

$$\therefore EL = 2BL.$$

(ii)



Proof : Draw seg PA,

seg PL, seg PN, seg PB,

seg PC and seg PM

$PL = PM = PN = r$... (Radii of the same circle)

$PM \perp BC, PL \perp AB, PN \perp AC,$... (Tangent theorem)

$$\therefore A(\triangle PBC) = \frac{1}{2} \times BC \times PM$$

$$A(\triangle PBC) = \frac{1}{2} \times BC \times r \quad \dots (1)$$

$$A(\triangle PAC) = \frac{1}{2} \times AC \times r \quad \dots (2)$$

$$A(\triangle PAB) = \frac{1}{2} \times AB \times r \quad \dots (3)$$

$\therefore A(\triangle ABC) = A(\triangle PBC) + A(\triangle PAC) + A(\triangle PAB)$... [From (1), (2) and (3)]

$$= \frac{1}{2} \times BC \times r + \frac{1}{2} \times AC \times r + \frac{1}{2} \times AB \times r$$

$$= \frac{1}{2}(BC + AC + AB) \times r$$

$$A(\triangle ABC) = \frac{1}{2}(AB + BC + AC) \times r.$$

(iii) For metallic cuboid,

length (l) = 44 cm

breadth (b) = 42 cm

height (h) = 21 cm

For the sphere,

Let the radius be r .

Volume of the sphere = Volume of metallic cuboid

$$\frac{4}{3}\pi r^3 = l \times b \times h$$

$$\therefore \frac{4}{3} \times \frac{22}{7} \times r^3 = 44 \times 42 \times 21$$

$$\therefore r^3 = \frac{44 \times 42 \times 21 \times 3 \times 7}{4 \times 22}$$

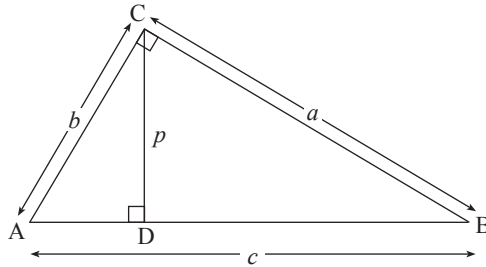
$$\therefore r^3 = 9261$$

$$\therefore r = 21 \text{ cm}$$

$$\begin{aligned} \text{Surface area of sphere} &= 4\pi r^2 \\ &= 4 \times \frac{22}{7} \times 21 \times 21 \\ &= 5544 \text{ cm}^2 \end{aligned}$$

Ans. Surface area of the sphere is **5544 cm²**.

Q. 5. (i)



Proof : $\triangle DAC \sim \triangle CAB$... (Similarity of right angled triangles)

$$\therefore \frac{CD}{BC} = \frac{AC}{AB} \quad \dots \text{ (c.s.s.t)}$$

$$\therefore \frac{p}{a} = \frac{b}{c} \quad \therefore p = \frac{ab}{c}$$

$$\therefore p^2 = \frac{a^2 b^2}{c^2} \quad \dots \text{ (Squaring both the sides)}$$

$$\therefore \frac{1}{p^2} = \frac{c^2}{a^2 b^2} \quad \dots \text{ (By invertendo) } \dots (1)$$

In right angled $\triangle CAB$, by Pythagoras theorem,

$$AB^2 = BC^2 + AC^2$$

$$\therefore c^2 = a^2 + b^2 \quad \dots (2)$$

From (1) and (2),

$$\begin{aligned} \frac{1}{p^2} &= \frac{a^2 + b^2}{a^2 b^2} \\ &= \frac{a^2}{a^2 b^2} + \frac{b^2}{a^2 b^2} \end{aligned}$$

$$\therefore \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

$$\text{i.e. } \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$

(ii) Analysis :

For $\triangle ABC$, the lengths of three sides are known.

$\therefore \triangle ABC$ can be constructed.

$\triangle ABC \sim \triangle LMN$

$\therefore \frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN} \dots$ (Corresponding sides of similar triangles are in proportion)

$$\therefore \frac{5.5}{LM} = \frac{6}{MN} = \frac{4.5}{LN} = \frac{5}{4}$$

$\therefore \frac{5.5}{LM} = \frac{5}{4}$	$\frac{6}{MN} = \frac{5}{4}$	$\frac{4.5}{LN} = \frac{5}{4}$
$\therefore LM = \frac{5.5 \times 4}{5}$	$\therefore MN = \frac{6 \times 4}{5}$	$\therefore LN = \frac{4.5 \times 4}{5}$
$\therefore LM = 1.1 \times 4$	$\therefore MN = \frac{24}{5}$	$\therefore LN = \frac{18}{5}$
$\therefore LM = 4.4 \text{ cm}$	$\therefore MN = 4.8 \text{ cm}$	$\therefore LN = 3.6 \text{ cm}$

$\therefore \triangle LMN$ can be constructed.

Ans.

