

SOLUTION : PRACTICE QUESTION PAPER 3

- Q. 1. (A)** (i) (C)
 (ii) (D)
 (iii) (B)
 (iv) (B)

Q. 1. (A) Explanation to the answers to MCQs in this question has been given below for students' guidance. Please note that, **Students are not expected to write the explanation in the examination.**

Explanations :

(i) (C) [Ratio of areas of two similar triangle = ratio of square of their corresponding sides.]

$$\therefore \frac{A_1}{A_2} = \frac{(S_1)^2}{(S_2)^2}$$

(ii) (D) [Side of square = $\frac{1}{\sqrt{2}} \times$ diagonal, perimeter = 4 \times side]

(iii) (B) Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(iv) (B)
$$\left[\begin{array}{l} C = 2\pi r, \\ A = \pi r^2 \\ \therefore \frac{2\pi r}{\pi r^2} = \frac{2}{7} \quad \therefore r = 7 \\ \quad \quad \quad \quad \quad \therefore d = 14 \end{array} \right]$$

Q. 1. (B) (i) **Solution :** Volume of the cylinder = 600 cm³ ... (Given)

The cylinder and cone have equal base radii and perpendicular heights.

$$\begin{aligned} \therefore \text{Volume of the cone} &= \frac{1}{3} \times \text{Volume of cylinder} \\ &= \frac{1}{3} \times 600 \\ &= 200 \text{ cm}^3 \end{aligned}$$

Ans. Volume of the cone is **200 cm³**.

(ii) **Solution :** Slope of X-axis is 0

Parallel lines have equal slopes.

Ans. Thus slope of line parallel to X-axis is **0**.

(iii) **Solution :** In $\triangle QPR$,

$$\left. \begin{array}{l} \angle QPR = 90^\circ \\ \angle PQR = 60^\circ \\ \angle PRQ = 30^\circ \end{array} \right\} \dots \text{ (Given)}$$

$\therefore \triangle QPR$ is a $30^\circ - 60^\circ - 90^\circ$ triangle

\therefore By $30^\circ - 60^\circ - 90^\circ$ triangle theorem,

$$PQ = \frac{1}{2} QR \quad \dots \text{ (Side opposite to } 30^\circ \text{)}$$

$$\therefore 7 = \frac{1}{2} QR$$

$$\therefore QR = 7 \times 2$$

$$\therefore QR = 14 \text{ cm}$$

Ans. QR = 14 cm.

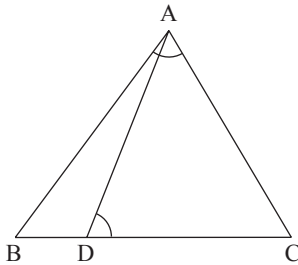
(iv) **Solution :** $\frac{AX}{XB} = \frac{2}{4} = \frac{1}{2}$ and $\frac{AY}{YC} = \frac{3}{6} = \frac{1}{2}$

$$\therefore \frac{AX}{XB} = \frac{AY}{YC}$$

\therefore line $XY \parallel$ side BC \dots (By converse of basic proportionality theorem)

Ans. By converse of basic proportionality theorem, line $XY \parallel$ side BC .

Q. 2. (A) (i) Activity :



In $\triangle BAC$ and $\triangle ADC$,

$$\angle BAC \cong \angle ADC \quad \dots \text{ (Given)}$$

$$\angle ACB \cong \boxed{\angle DCA} \quad \dots \text{ (Common angle)}$$

$$\therefore \triangle BAC \sim \triangle ADC \quad \dots \text{ (} \boxed{\text{AA}} \text{ test of similarity)}$$

$$\therefore \frac{CA}{\boxed{CD}} = \frac{CB}{\boxed{CA}} \quad \dots \text{ (Corresponding sides of similar triangles)}$$

$$\therefore CA^2 = CB \times CD.$$

(ii) Activity :

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$1 + \left(\frac{40}{9}\right)^2 = \operatorname{cosec}^2 \theta \quad \dots \text{ (Substituting the values)}$$

$$\therefore 1 + \frac{1600}{81} = \operatorname{cosec}^2 \theta$$

$$\therefore \operatorname{cosec}^2 \theta = \frac{1681}{81}$$

$$\therefore \operatorname{cosec} \theta = \frac{41}{9}$$

$$\therefore \sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$\therefore \sin \theta = \frac{9}{41}$$

(iii) Activity :

For cylindrical bucket :

diameter = 28 cm

radius (r) = 14 cm

height (h) = 20 cm

For sand cone :

height (h_1) = 14 cm

Let the radius be r_1

base area = πr_1^2

Here, volume of cylindrical bucket = Volume of the sand cone

$$\pi r^2 h = \frac{1}{3} \pi r_1^2 h_1$$

$$\therefore \frac{22}{7} \times 14 \times 14 \times \boxed{20} = \frac{1}{3} \pi r_1^2 \times \boxed{14}$$

$$\therefore \pi r_1^2 = \frac{22 \times 14 \times 14 \times \boxed{20} \times 3}{7 \times 14}$$

$$\therefore \pi r_1^2 = \text{base area of cone} = \boxed{2640} \text{ cm}^2.$$

Q. 2. (B) (i) Solution :

Let radius of first circle is r_1 and radius of second circle is r_2 .

Both the circles touching internally.

$$\therefore |r_1 - r_2| = 10$$

$$\therefore r_1 - r_2 = 10 \quad \dots (1) \quad \text{or} \quad r_1 - r_2 = -10 \quad \dots (2)$$

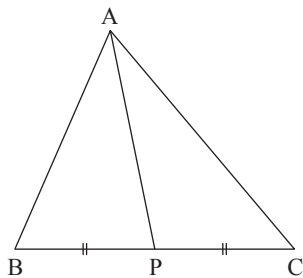
$$\therefore 15 - r_2 = 10 \quad \dots \text{ [From (1)]} \quad \left| \quad 15 - r_2 = -10 \right.$$

$$\therefore 15 - 10 = r_2 \quad \left| \quad \therefore 15 + 10 = r_2 \right.$$

$$\therefore r_2 = 5 \text{ cm} \quad \left| \quad \therefore r_2 = 25 \text{ cm} \right.$$

Ans. Radius and second circle is **5 cm** or **25 cm**.

(ii)



$$BC = 18 \quad \dots \text{ (Given)}$$

$$BP = \frac{1}{2} BC \quad \dots \text{ (P is the midpoint of seg BC)}$$

$$\therefore BP = \frac{1}{2} \times 18$$

$$\therefore BP = 9$$

In $\triangle ABC$,

seg AP is the median ... (Given)

\therefore by Apollonius theorem,

$$AB^2 + AC^2 = 2AP^2 + 2BP^2$$

$$\therefore 260 = 2AP^2 + 2(9)^2$$

$$\therefore 260 = 2AP^2 + 162$$

$$\therefore 2AP^2 = 260 - 162$$

$$\therefore 2AP^2 = 98$$

$$\therefore AP^2 = \frac{98}{2}$$

$$\therefore AP^2 = 49$$

$$\therefore AP = 7 \quad \dots \text{ (Taking square roots of both the sides)}$$

Ans. $AP = 7$.

(iii) Solution : Let $A(-7, 6)$, $B(2, -2)$ and $C(8, 5)$.

$$\text{Let } A(-7, 6) \equiv (x_1, y_1)$$

$$B(2, -2) \equiv (x_2, y_2)$$

$$C(8, 5) \equiv (x_3, y_3)$$

Let $G(x, y)$ be the centroid.

By centroid formula,

$$x = \frac{x_1 + x_2 + x_3}{3}$$

$$\therefore x = \frac{-7 + 2 + 8}{3}$$

$$\therefore x = \frac{3}{3}$$

$$\therefore x = 1$$

$$y = \frac{y_1 + y_2 + y_3}{3}$$

$$\therefore y = \frac{6 + (-2) + 5}{3}$$

$$\therefore y = \frac{9}{3}$$

$$\therefore y = 3$$

Ans. The coordinates of the centroid are **(1, 3)**.

(iv) **Proof :**

$$\begin{aligned}\text{LHS} &= \frac{\sin^2 \theta}{\cos \theta} + \cos \theta \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \\ &= \frac{1}{\cos \theta} && [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= \sec \theta \\ &= \text{RHS} \\ \therefore \frac{\sin^2 \theta}{\cos \theta} + \cos \theta &= \sec \theta.\end{aligned}$$

(v) **Solution :** Diameter of a sphere = 6 cm

its radius (r) = 3 cm

$$\begin{aligned}\text{Volume of the sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times 3.14 \times 3 \times 3 \times 3 \\ &= 113.04 \text{ cm}^3\end{aligned}$$

Ans. Volume of the sphere is **113.04 cm³**.

Q. 3. (A) (i) Activity :

In right angled $\triangle ABE$,

by Pythagoras theorem,

$$AE^2 = AB^2 + \boxed{BE^2} \quad \dots (1)$$

In right angled $\triangle DBC$,

by Pythagoras theorem,

$$CD^2 = \boxed{DB^2} + BC^2 \quad \dots (2)$$

In right angled $\triangle ABC$,

by Pythagoras theorem,

$$AC^2 = AB^2 + \boxed{BC^2} \quad \dots (3)$$

In right angled $\triangle DBE$,

by Pythagoras theorem,

$$DE^2 = BD^2 + \boxed{BE^2} \quad \dots (4)$$

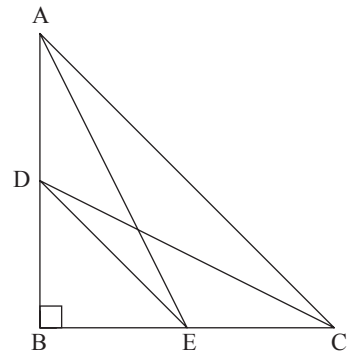
Adding (1) and (2), we get

$$AE^2 + CD^2 = AB^2 + BE^2 + BD^2 + BC^2$$

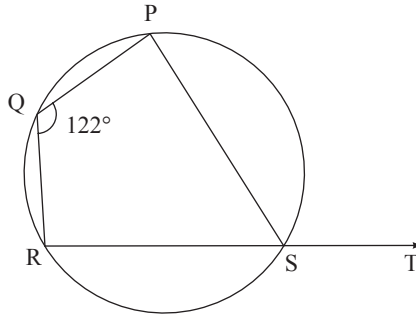
$$\therefore AE^2 + CD^2 = (AB^2 + BC^2) + (BD^2 + BE^2) \quad \dots (5)$$

Substituting the values of (3) and (4) in (5), we get

$$AE^2 + CD^2 = \boxed{AC^2} + \boxed{DE^2}.$$



(ii)



Activity :

□ PQRS is cyclic.

Opposite angles of a cyclic quadrilateral are **supplementary**

$$\angle PQR = 122^\circ \quad \dots \text{ (Given)}$$

$$\text{and } \angle PQR + \angle PSR = \mathbf{180^\circ}$$

$$\therefore 122^\circ + \angle PSR = \mathbf{180^\circ}$$

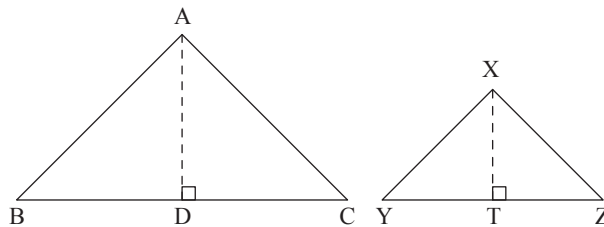
$$\therefore \angle PSR = \mathbf{58^\circ}$$

$\therefore \angle PST$ is an exterior angle of cyclic quadrilateral PQRS.

$$\angle PST = \angle \mathbf{PQR} \quad \dots \text{ (Corollary of cyclic quadrilateral theorem)}$$

$$\therefore \angle PST = \mathbf{122^\circ}$$

Q. 3. (B) (i)



Given : $\triangle ABC \sim \triangle XYZ$

$$\text{To prove : } \frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{AB^2}{XY^2} = \frac{BC^2}{YZ^2} = \frac{AC^2}{XZ^2}$$

Construction : Draw seg $AD \perp$ side BC such that B–D–C and seg $XT \perp$ side YZ such that Y–T–Z.

Proof : Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.

$$\therefore \frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{BC \times AD}{YZ \times XT}$$

$$\therefore \frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{BC}{YZ} \times \frac{AD}{XT} \quad \dots (1)$$

$$\triangle ABC \sim \triangle XYZ \quad \dots \text{ (Given)}$$

$$\therefore \angle B \cong \angle Y \quad \dots \text{ (Corresponding angles of similar triangles) } \dots (2)$$

and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$... (Corresponding sides of similar triangles are in proportion) ... (3)

In $\triangle ABD$ and $\triangle XYT$,

$\angle ABD \cong \angle XYT$... [From (2)]

$\angle ADB \cong \angle XTY$... (Each measures 90°)

$\therefore \triangle ABD \sim \triangle XYT$... (AA test of similarity)

$\therefore \frac{AB}{XY} = \frac{AD}{XT}$... (Corresponding sides of similar triangles are in proportion) ... (4)

\therefore from (3) and (4), we get

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AD}{XT}$$

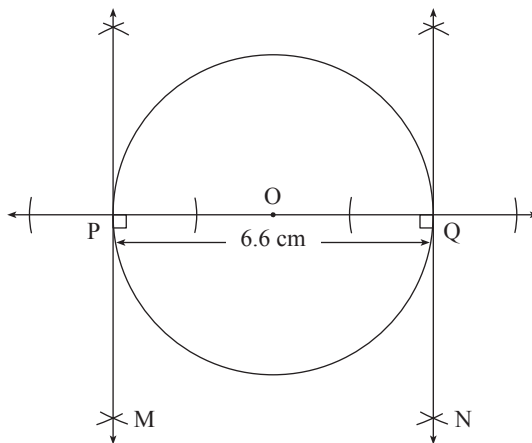
$\therefore \frac{BC}{YZ} = \frac{AD}{XT}$... (5)

$\frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{BC}{YZ} \times \frac{BC}{YZ}$... [From (1) and (5)]

$\therefore \frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{BC^2}{YZ^2}$... (6)

$\therefore \frac{A(\triangle ABC)}{A(\triangle XYZ)} = \frac{AB^2}{XY^2} = \frac{BC^2}{YZ^2} = \frac{AC^2}{XZ^2}$... [From (1) and (6)]

(ii)



The tangents drawn at points P and Q are parallel.

(iii) **Solution :** A(- 1, 7) and B(4, - 3).

Let the coordinates of point P be (x, y).

Suppose coordinates of point A are (x_1, y_1) and B are (x_2, y_2) then $x_1 = - 1, y_1 = 7,$
 $x_2 = 4$ and $y_2 = - 3.$

P divides AB in the ratio 2 : 3.

$\therefore m : n = 2 : 3$

By section formula,

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$\therefore x = \frac{2(4) + 3(-1)}{2+3}$$

$$\therefore x = \frac{8-3}{5}$$

$$\therefore x = \frac{5}{5}$$

$$\therefore x = 1$$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$\therefore y = \frac{2(-3) + 3(7)}{2+3}$$

$$\therefore y = \frac{-6+21}{5}$$

$$\therefore y = \frac{15}{5}$$

$$\therefore y = 3$$

Ans. P(1, 3) divides the line segment joining the points A(−1, 7) and B(4, −3) in the ratio 2 : 3.

(iv) Solution :

The radius (r) of the circle = OP = 10 cm.

$m(\text{arc PQR}) = \theta = 60^\circ$.

The shaded region is segment PQR,

$$A(\text{segment PQR}) = r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2} \right]$$

$$= 10^2 \left[\frac{3.14 \times 60}{360} - \frac{\sin\theta}{2} \right]$$

$$= 100 \left[\frac{3.14}{6} - \frac{\sqrt{3}}{2 \times 2} \right] \quad \dots \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2} \right]$$

$$= 100 \times \left[\frac{3.14}{6} - \frac{1.73}{4} \right] \quad \dots (\because \sqrt{3} = 1.73)$$

$$= 100 \left[\frac{3.14 \times 2 - 1.73 \times 3}{12} \right]$$

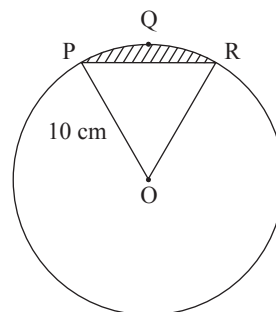
$$= 100 \left[\frac{6.28 - 5.19}{12} \right]$$

$$= 100 \times \frac{1.09}{12}$$

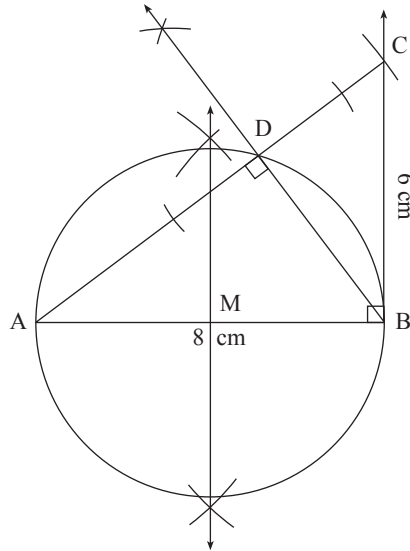
$$= \frac{109}{12}$$

$$\approx 9.08 \text{ cm}^2$$

Ans. Area of the shaded region is **9.08 cm²**.



Q. 4. (i)



Proof : A circle passes through the points A, D and B, $\angle ADB = 90^\circ$

So arc ADB is a semicircle.

Hence AB is the diameter.

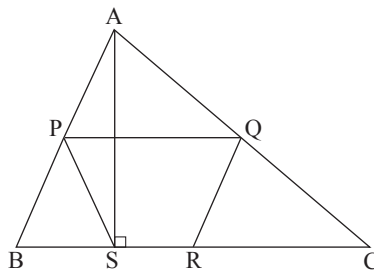
M is the midpoint of the diameter and hence the centre of the circle.

\therefore MB is the radius.

CB is perpendicular to radius MB at its outer end B.

Thus CB is tangent to the circle ... (Converse of tangent theorem)

(ii)



Proof :

In $\triangle ABC$, P and Q are the midpoints of side AB and side AC respectively.

$\therefore PQ \parallel BC$... (Midpoint theorem)

$\therefore PQ \parallel BR$... (B – R – C) ... (1)

Similarly, $QR \parallel BP$... (2)

\therefore from (1) and (2) $\square PQRB$ is a parallelogram.

$\angle B = \angle PQR$... (Opposite angles of parallelogram are equal) ... (3)

In $\triangle ASB$,

$\angle ASB = 90^\circ$... (Given)

Seg SP is median drawn to hypotenuse AB.

$$SP = \frac{1}{2} AB \quad \dots \left[\begin{array}{l} \text{In a right angled triangle, the median drawn} \\ \text{to the hypotenuse is half the hypotenuse} \end{array} \right] \quad \dots (4)$$

$$\text{also } PB = \frac{1}{2} AB \quad \dots (\text{P is the midpoint of AB}) \quad \dots (5)$$

In $\triangle PBS$,

$$SP = PB \quad \dots [\text{From (4) and (5)}]$$

$$\therefore \angle B = \angle PSB \quad \dots (\text{Isosceles triangle theorem}) \quad \dots (6)$$

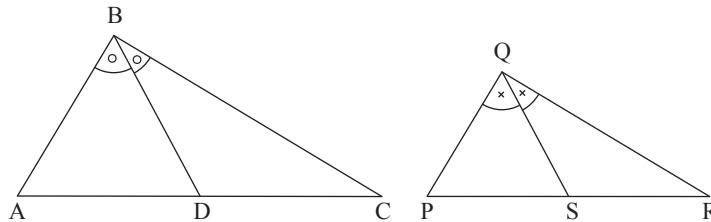
$$\therefore \angle PSB = \angle PQR \quad \dots [\text{From (3) and (6)}] \quad \dots (7)$$

$$\angle PSB + \angle PSR = 180^\circ \quad \dots (\text{Linear pair of angles})$$

$$\therefore \angle PQR + \angle PSR = 180^\circ \quad \dots [\text{From (7)}]$$

$$\therefore \square PQRS \text{ is cyclic} \quad \dots [\text{A quadrilateral is cyclic, if its opposite angles are supplementary}]$$

(iii)



Proof :

$$\frac{AD}{PS} = \frac{DC}{SR} \quad \dots (\text{Given})$$

\therefore by alternendo,

$$\frac{AD}{DC} = \frac{PS}{SR} \quad \dots (1)$$

In $\triangle ABC$,

BD bisects $\angle ABC$

$$\frac{AB}{BC} = \frac{AD}{DC} \quad \dots (2) [\text{Theorem of angle bisector of a triangle}]$$

In $\triangle PQR$,

QS bisects $\angle PQR$

$$\therefore \frac{PQ}{QR} = \frac{PS}{SR} \quad \dots (3) [\text{Theorem of angle bisector of a triangle}]$$

\therefore from (1), (2) and (3), we get

$$\frac{AB}{BC} = \frac{PQ}{QR}$$

\therefore by alternendo, we get

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad \dots (4)$$

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad \dots \text{ [From (4)]}$$

$$\angle ABC \cong \angle PQR \quad \dots \text{ (Given)}$$

$$\therefore \triangle ABC \sim \triangle PQR \quad \text{(SAS test of similarity)}$$

Q. 5. (i) **Solution :**

$$\tan \theta + \frac{1}{\tan \theta} = 2 \quad \dots \text{ (Given)}$$

Squaring both the sides, we get

$$\left(\tan \theta + \frac{1}{\tan \theta} \right)^2 = 2^2$$

$$\tan^2 \theta + 2 \times \cancel{\tan \theta} \times \frac{1}{\cancel{\tan \theta}} + \frac{1}{\tan^2 \theta} = 4$$

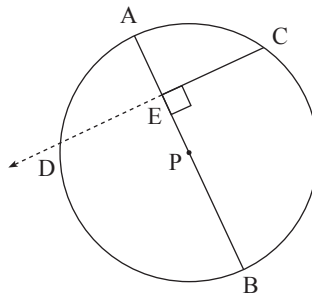
$$\tan^2 \theta + 2 + \frac{1}{\tan^2 \theta} = 4$$

$$\tan^2 \theta + \frac{1}{\tan^2 \theta} = 4 - 2$$

$$\tan^2 \theta + \frac{1}{\tan^2 \theta} = 2$$

Ans. The value of $\tan^2 \theta + \frac{1}{\tan^2 \theta}$ is **2**.

(ii) (a)



(b) Seg $PE \perp$ chord CD .

Perpendicular drawn from the centre of the circle to the chord bisects the chord

$$\therefore CE = ED \quad \dots \text{ (i)}$$

(c) Chords AB and CD intersect at point E inside the circle

$$\therefore CE \times ED = AE \times EB \quad \dots \text{ (ii)}$$

(d) $CE \times ED = AE \times EB \quad \dots \text{ [From (ii)]}$

$$\therefore CE \times CE = AE \times EB \quad \dots \text{ [From (i)]}$$

$$\therefore CE^2 = AE \times EB$$

$\therefore CE$ is the geometric mean of AE and EB .